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THESIS

TARGET VULNERABILITY
TO AIR DEFENSE WEAPONS

by

Bruce Edward Reinard

December 1984

Thesis Advisor: R. E. Ball

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aircraft from an externally detonating warhead. The section on target vulnerability explains the methodology used for identifying critical components and conducting a vulnerability assessment, and leads to the effects of fragments and penetrators striking an aircraft.

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Target Vulnerability to Air Defense Weapons

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Submitted in partial fulfillment of the
requirements for the degree of

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December 1984

ABSTRACT

This thesis is intended to become a portion of the textbook utilized in the course entitled "Warheads and Lethality" (AE-3705). This portion of the text includes an unclassified discussion of a target's susceptibility to an externally detonating HE warhead and a target's vulnerability. In particular, the section on target susceptibility leads to the development of the number of fragments which strike a target aircraft from an externally detonating warhead. The section on target vulnerability explains the methodology used for identifying critical components and conducting a vulnerability assessment, and leads to the effects of fragments and penetrators striking an aircraft.

TABLE OF CONTENTS

I. SUSCEPTIBILITY TO EXTERNALLY DETONATING FRAGMENTATION WARHEADS 11

A. INTRODUCTION 11

B. ENCOUNTER SCENARIO 11

1. Warhead Dynamic Spray Angles and Velocities 12

2. Fragment Spray Density 14

3. Missile Miss Distance 16

4. Fragment Miss Distance 22

5. Fragment Impacts on the Target 27

II. TARGET VULNERABILITY 34

A. INTRODUCTION 34

B. IDENTIFICATION OF CRITICAL COMPONENTS 34

1. Aircraft Kill Levels 35

2. Aircraft Description 37

3. Critical Component Analysis 38

C. VULNERABILITY ASSESSMENT 64

1. Defining a Vulnerability Assessment . . . 64

2. Vulnerability Measures 66

3. General Requirements 69

D. VULNERABILITY TO EXTERNALLY DETONATING WARHEADS 77

1. Effect of Fragments/Penetrators Striking an Aircraft 77

2. Blast 121

3. Endgame Analysis 123

E. VULNERABILITY TO INTERNALLY DETONATING HE WARHEADS 128

F. VULNERABILITY TO LASERS 132
G. COMPUTER PROGRAMS FOR VULNERABILITY ASSESSMENT 133
1. Shotline Generators 134
2. Vulnerable Area Routines 136
3. Internal Burst Programs 142
4. Endgame Programs 142

BIBLIOGRAPHY 147

INITIAL DISTRIBUTION LIST 148

LIST OF TABLES

1. System Damage-Caused Failure Modes 51
2. Vulnerability Assessment Variable Definitions . . 79
3. Nonredundant Mcdel Withcut Overlap With Mutually Exclusive Kill Modes 86
4. Nonredundant Model Without Overlap With Mutually Inclusive Kill Modes 87
5. Vulnerable Area Computation for Nonredundant Mcdel With Overlap and Nc Engine Fire 91
6. Vulnerable Area Computation for Nonredundant Model With Overlap and an Engine Fire 92
7. Redundant Aircraft Model Without Overlap 95
8. Redundant Aircraft Model With Overlap 100
9. Computation of the State Transition Matrix . . . 110
10. A Comparison of Aircraft Kill Probabilities . . 115
11. An Example Computation fcr $P_{K/D}$ 127

LIST OF FIGURES

1.1 Typical Warhead Depicting Dynamic Spray Angles and Velocities 13
1.2 Sample Encounter with a Horizontally Moving Aircraft Depicting the Fragment Spray Density . . . 15
1.3 Typical Encounter Scenario in the Global System . . 17
1.4 Global System Encounter Scenario Using Vector Notation 18
1.5 Encounter Scenario in the Local System 21
1.6 Fragment Miss Distance Depicted in Global Coordinate System 23
1.7 Fragment Miss Distance Using Vector Notation in Global Coordinate System 24
1.8 Fragment Miss Distance Depicted in Local Coordinate System 25
1.9 Fragment Impacts on Aircraft in Local System . . . 28
1.10 Fragment Impacts on Aircraft in Global System . . . 29
1.11 Effect of Varying Detonation Distance on Fragments Impacting on an Aircraft 31
1.12 Frontal View of Fragments Impacting an Aircraft . . 32
2.1 Some Essential Functions and Mission Phases for an Attack Helicopter 42
2.2 System-Essential Function Relationships 43
2.3 Subsystem Functions- Essential Function Relationships 44
2.4 Sample FMEA Summary Format 46
2.5 A Sample DMEA Matrix 48
2.6 A Sample Disablement Diagram 49
2.7 A Generic Fault Tree Diagram 59

2.8a Portion of a Damage Tree Diagram for a Twin Engine Aircraft 60
2.8b Portion of a Damage Tree Diagram Cont'd 61
2.8c Portion of a Damage Tree Diagram Cont'd 62
2.9 Sample Kill Tree for a Two Engine Two Pilot Helicopter 65
2.10 Six Aircraft Aspects for a Minimum Assessment . . . 67
2.11 26 Aircraft Aspects for a Detailed Analysis 68
2.12 Typical $P_{k/h}$ Data for a Flight Control Rod 73
2.13 Engine $P_{k/h}$ Data 74
2.14 Typical Aircraft Encounter With An Externally Detonating HE Warhead 78
2.15 Nonredundant Aircraft Model With No Overlap 83
2.16 Nonredundant Aircraft Model With Overlap 89
2.17 Redundant Aircraft Model With No Overlap 94
2.18 Redundant Aircraft Model With Overlap 98
2.19 First Hit Nonredundant Kill Tree 103
2.20 Second Hit Nonredundant Kill Tree 104
2.21 First Hit Nonredundant Kill Tree Example 104
2.22 Second Hit Nonredundant Kill Tree Example 105
2.23 First Hit Redundant Kill Tree Example 107
2.24 Second Hit After First Hit on Engine 1 108
2.25 Probability of Kill for the Redundant Aircraft Model vs the Number of Hits 114
2.26 Vulnerable Area vs Number of Hits 118
2.27 Total Aircraft Single Hit Vulnerable Form 120
2.28 Sample Component Vulnerable Area Form 121
2.29 Typical External Blast Vulnerability Data Presentation 123
2.30 Typical External Blast K-Kill Contour for Various Weights of Uncased Pentolite at Sea Level 124
2.31 Typical Probability of Kill Given A Detonation Function 126

2.32 The Expanded Area Approach 129
2.33 The Point Burst Approach 130
2.34 Example Grid for the Shotline 135
2.35 Combinatorial Geometry Model of an Aircraft . . . 137
2.36 External View of Aircraft Using Triangular Surface Patch Method 138
2.37 Internal Components Modeled Using Triangular Patch Method 139

I. SUSCEPTIBILITY TO EXTERNALLY DETONATING FRAGMENTATION WARHEADS

A. INTRODUCTION

Susceptibility refers to the inability of a target to avoid being damaged in the pursuit of its mission. For aircraft, susceptibility to an externally detonating warhead refers to an aircraft's probability of being hit. Susceptibility, like vulnerability, is good. The level or degree of susceptibility of an aircraft in an encounter with a threat is dependent upon three major factors, the encounter scenario, the threat, and the aircraft. The encounter scenario includes the missile and aircraft positions, velocity vectors, respective attitudes, a determination of the warhead's fragment dynamic spray angles, a determination of the missile miss distance, and the determination of how many fragments or penetrators strike the aircraft. The important features of the threat are its characteristics, its operations, and its lethality. The important aircraft features are the aircraft detectable signatures, countermeasures, performance capabilities, and self-protection armament.

B. ENCOUNTER SCENARIO

The encounter scenario takes into account the missile's flight path and the target's flight path to allow calculation of the missile miss distance, fragment miss distance, the warhead's dynamic fragment spray angles and velocities, and the number of fragments which strike the target.

1. Warhead Dynamic Spray Angles and Velocities

When a warhead detonates in the vicinity of an aircraft, the fragments or peretrators are usually ejected uniformly around the missile axis and propagate outward in a divergent spherical-like spray pattern at a velocity that is the vector sum of the initial fragment ejection velocity from a static warhead detonation and the missile velocity. The fragment at the front of the warhead is assumed to propagate outward at the leading spray angle, and the fragment at the tail end of the warhead is assumed to propagate along trajectory at the trailing spray angle. All other fragment trajectories lie between these two spray angles and constitute the fragment spray. As the aircraft moves in space, the fragments propagate outward and eventually some of the fragments may strike the aircraft. Whether or not any of the fragments strike the aircraft and where they hit depend upon the relative positions, velocities, and the attitude of the warhead and the aircraft at the time of detonation (encounter conditions) and the fragment static velocities and static spray angles. A sample warhead depicting static fragment spray angles and velocities, and dynamic fragment spray angles and velocities is depicted in Figure 1.1. Using the law of cosines and Figure 1.1, the dynamic fragment velocity, V_i, is given by:

$$V_i^2 = V_m^2 + V_s^2 - (2 \times V_m \times V_s \times \text{Cos}\,\beta) \qquad (1.1)$$

where V_m is the missile velocity and V_s is the static fragment velocity. Since the angle β is not known, Equation 1.1 must be rearranged to use the only known angle, which is the static fragment spray angle, α. Rearranging Equation 1.1, the dynamic fragment velocity is given by:

$$V_i^2 = V_m^2 + V_s^2 - [2 \times V_m \times V_s \times \text{Cos}(180-\alpha_i)] \qquad (1.2)$$

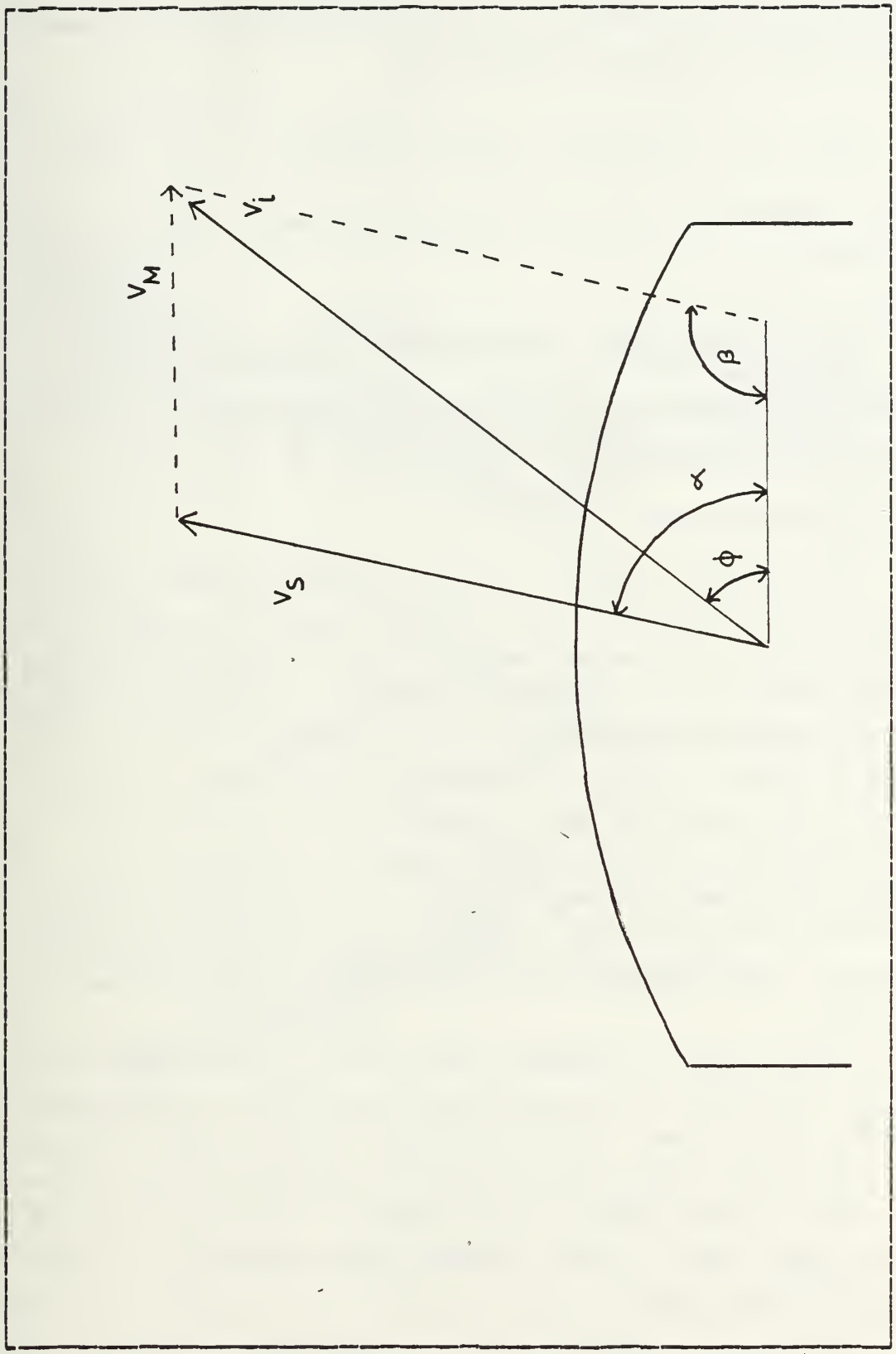


Figure 1.1 Typical Warhead Depicting Dynamic Spray Angles and Velocities

Again referring to Figure 1.1, the dynamic fragment velocity may be written in vector notation as:

$$V_i = [V_m + (V_s \times \cos\alpha_i)]i + [V_s \times \sin\alpha_i]j \qquad (1.3)$$

Solving Equation 1.3 for the dynamic fragment spray angle, ϕ_i, yields:

$$\phi_i = \tan^{-1}[(V_s \times \sin\alpha_i) / [V_m + (V_s \times \cos\alpha_i)]] \qquad (1.4)$$

Now that the dynamic fragment spray angles and velocities are known, the fragment spray density may be determined.

2. Fragment Spray Density

The damage inflicted on an aircraft depends on the number and the location of the fragment impacts, and on the terminal effects parameters such as the fragment mass and impact velocity. For this derivation of the fragment spray density, the following assumptions are made:

1. The fragments lie on a spherical segment whose center is at the center of the warhead.
2. The fragments emerge from the warhead in such a way that they remain on the surface of an expanding sphere.

An encounter scenario with a horizontally moving aircraft, and based on the assumptions stated above, is depicted in Figure 1.2.

For any given fragment spray zone, the density of fragments within that zone is simply the number of fragments contained in the zone divided by the surface area of the sphere contained within the conical angles defining the zone. The average number of fragments per unit area of fragment spray, known as the fragment spray density, ρ, is given by Equation 1.5.

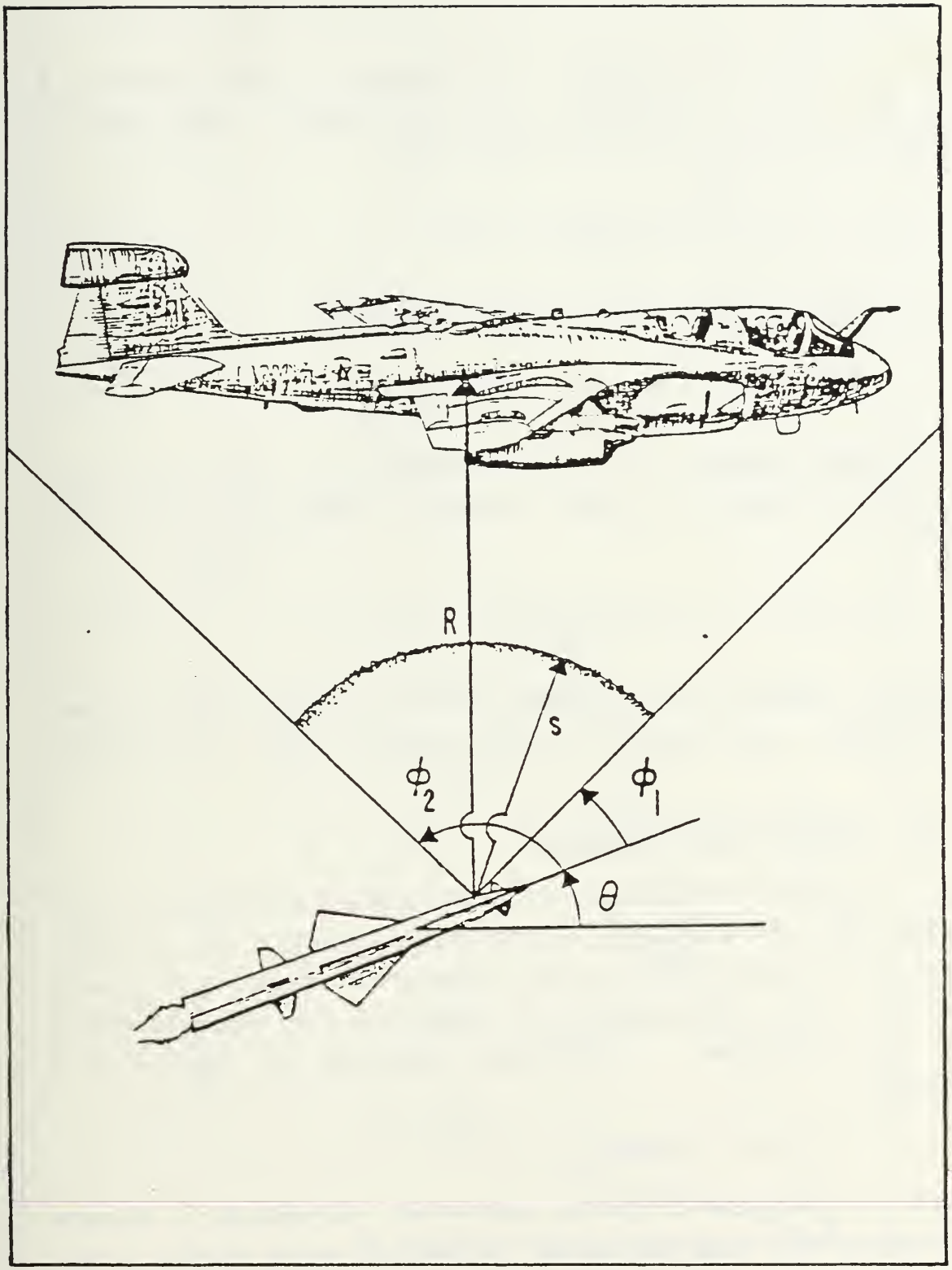


Figure 1.2 Sample Encounter with a Horizontally Moving Aircraft Depicting the Fragment Spray Density

$$\rho = N / A \qquad (1.5)$$

where N is the total number of fragments in the warhead and A is the area the fragments are spread over. Referring to Figure 1.2, the area, A, is given by:

$$A = 2 \times \pi \times \int_{\phi_1}^{\phi_2} (s \times \text{Sin}\, \phi) \times s \, d\phi \qquad (1.6)$$

Solving Equation 1.6 leads to the solution:

$$A = 2 \times \pi \times s^2 \times (\text{Cos}\, \phi_1 - \text{Cos}\, \phi_2) \qquad (1.7)$$

Substituting Equation 1.7 into Equation 1.5 yields the fragment spray density at some distance s from the detonation point:

$$\rho = N / [2 \times \pi \times s^2 \times (\text{Cos}\, \phi_1 - \text{Cos}\, \phi_2)] \qquad (1.8)$$

where the leading and trailing dynamic fragment spray angles are defined, with respect to the warhead axis, by Equation 1.4.

3. Missile Miss Distance

For the calculation of missile miss distance, it is assumed that the encounter is two dimensional, and that the target's velocity and the missile's velocity remain constant. Two approaches to calculate the miss distance will be presented. The first approach is the global approach.

a. Global Approach

Figure 1.3 depicts a typical encounter situation. Figure 1.4 depicts the same encounter situation using vector notation. Referring to Figure 1.4, and using vector addition, the missile miss distance, $\bar{s}$, is given by Equation 1.9.

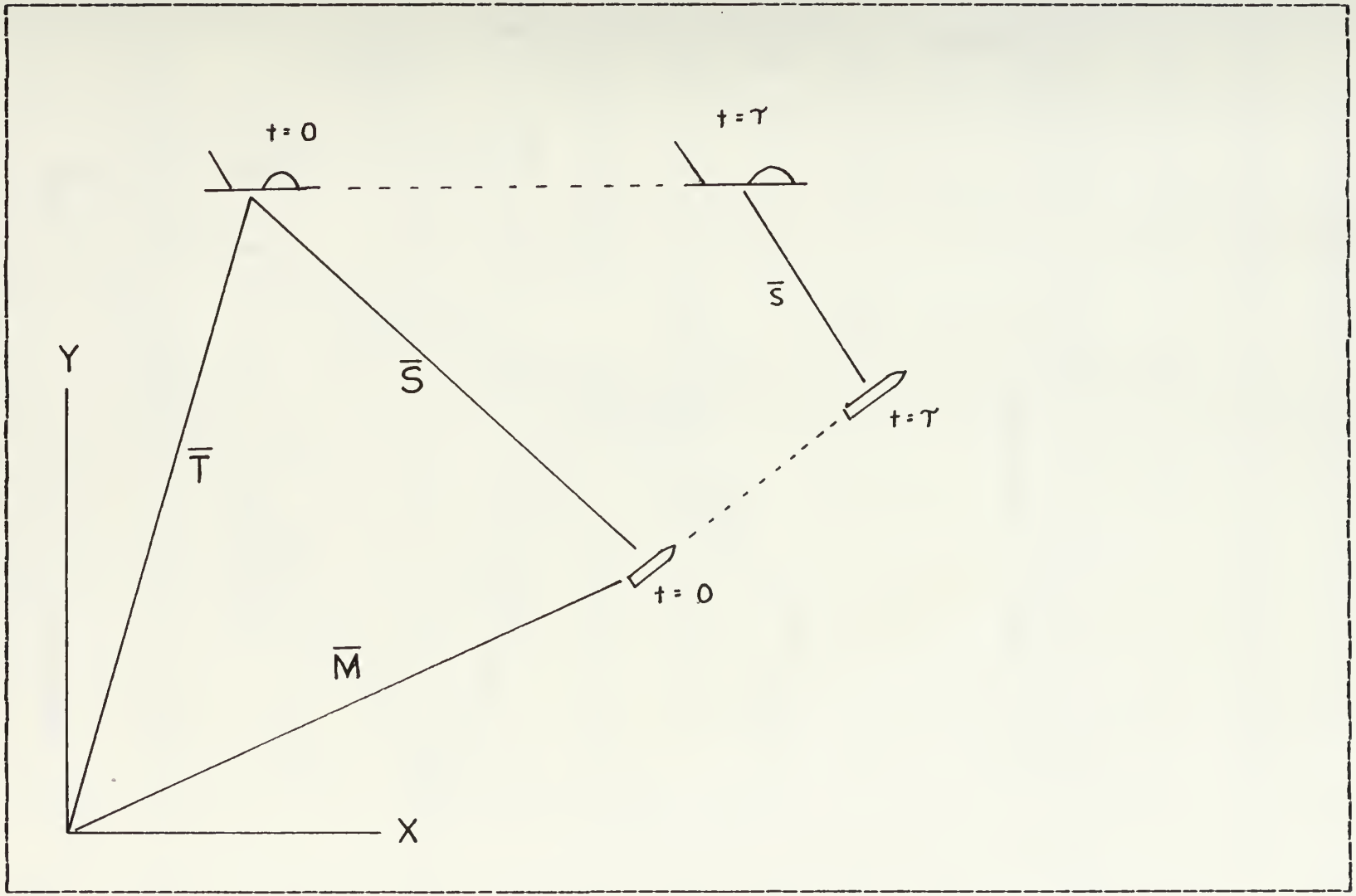


Figure 1.3 Typical Encounter Scenario in the Global System

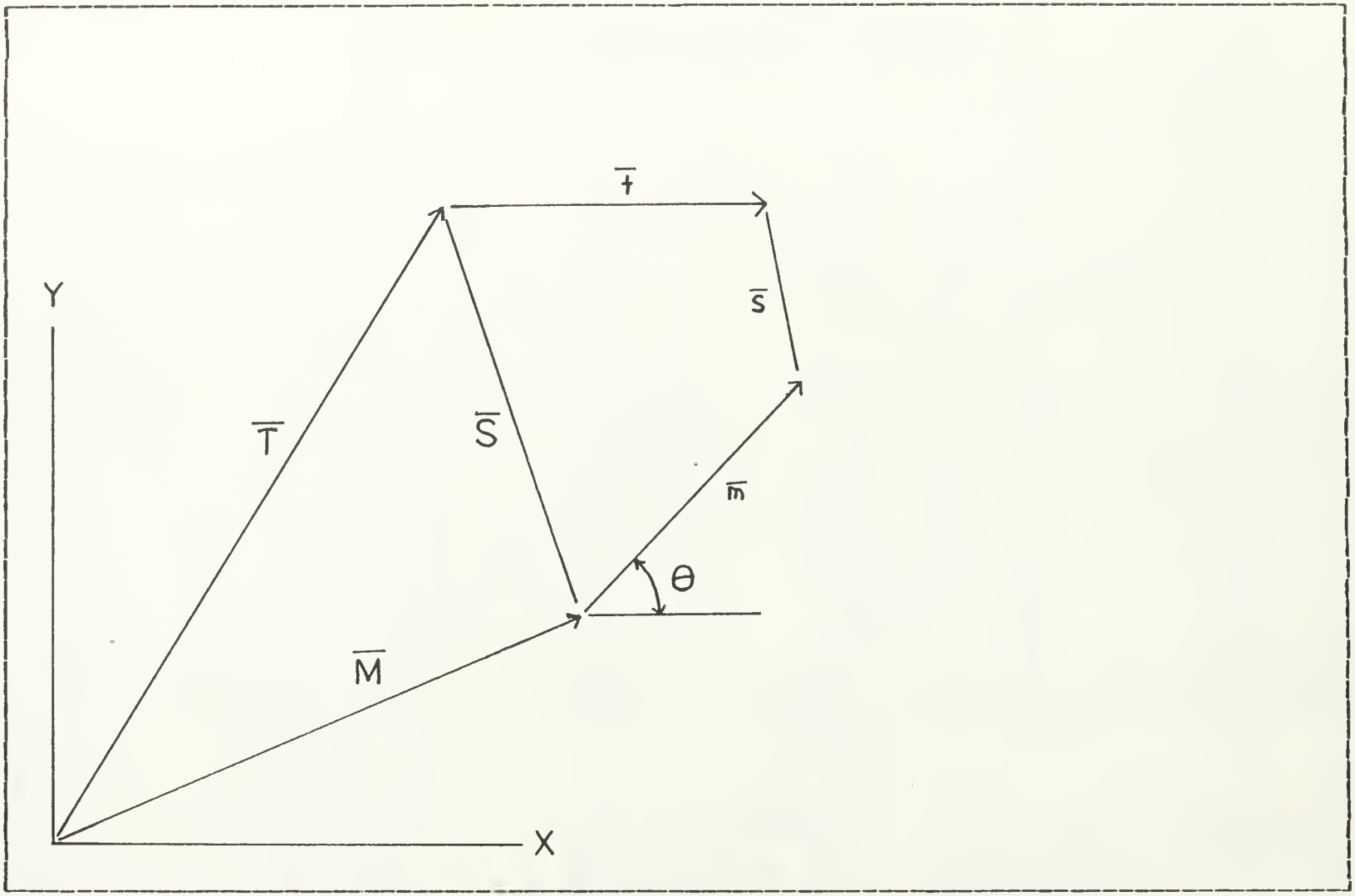


Figure 1.4 Global System Encounter Scenario Using Vector Notation

$$\bar{T} + \bar{t} = \bar{M} + \bar{m} + \bar{s} \tag{1.9}$$

Rearranging Equation 1.9 and solving for $\bar{s}$ leads to:

$$\bar{s} = (\bar{T} - \bar{M}) + (\bar{t} - \bar{m}) \tag{1.10}$$

The initial conditions (at time t = 0) are given by Equations 1.11 and 1.12. The conditions at some later time (t = τ) are given by Equations 1.13 and 1.14.

$$\bar{T} = (T_x)\hat{i} + (T_y)\hat{j} \tag{1.11}$$

$$\bar{M} = (M_x)\hat{i} + (M_y)\hat{j} \tag{1.12}$$

$$\bar{t} = (V_t \times \tau)\hat{i} \tag{1.13}$$

$$\bar{m} = (V_m \times \tau \times \text{Cos}\theta)\hat{i} + (V_m \times \tau \times \text{Sin}\theta)\hat{j} \tag{1.14}$$

Substituting these conditions back into Equation 1.10 gives:

$$\bar{s} = [(T_x - M_x)\hat{i} + (T_y - M_y)\hat{j}] + [((V_t \times \tau) - (V_m \times \tau \times \text{Cos}\theta))\hat{i} - (V_m \times \tau \times \text{Sin}\theta)\hat{j}] \tag{1.15}$$

Rearranging Equation 1.15 and combining similar components gives:

$$\bar{s} = [(T_x - M_x) + [\quad \times (V_t - (V_m \times \text{Cos}\theta))]]\hat{i} + [(T_y - M_y) - (V_m \times \tau \times \text{Sin}\theta)]\hat{j} \tag{1.16}$$

The missile miss distance is given by taking the magnitude of Equation 1.16 which is:

$$|\bar{s}| = [(T_x - M_x) + [\tau \times (V_t - (V_m \times \text{Cos}\theta))]]^2 + [(T_y - M_y) - (V_m \times \tau \times \text{Sin}\theta)]^2 \tag{1.17}$$

Of interest is the minimum miss distance. Taking the derivative of Equation 1.17 with respect to τ, and setting it equal to zero yields:

$$[(\{T_x - M_x\} \times \{V_t - (V \times \cos\theta)\}) - \{(T_y - M_y) \times (V_m \times \sin\theta)\}] + [\tau \times [\{V_t - (V_m \times \cos\theta)\}^2 + (V_m \times \sin\theta)^2]] = 0 \quad (1.18)$$

Solving Equation 1.18 for τ yields τ for the minimum miss distance which is:

$$= [((T_y - M_y) \times V_m \times \sin\theta) + ((T_x - M_x) \times V_m \times \cos\theta) - ((T_x - M_x) \times V_t)] / [(V_t - (V_m \times \cos\theta))^2 + (V_m \times \sin\theta)^2] \quad (1.19)$$

The minimum missile miss distance is then given by substituting the value of τ, obtained from Equation 1.19, into Equation 1.17.

b. Local Approach

The local approach is an alternative approach to the one described above. In the local approach, the target remains stationary and the target's velocity vector is superimposed on the missile's velocity vector. This geometry is depicted in Figure 1.5. From Figure 1.5, the velocity of the missile with the target's velocity superimposed, $\bar{V}_{mt}$, becomes:

$$\bar{V}_{mt} = (V_m \times \cos\theta)\hat{i} + (V_m \times \sin\theta)\hat{j} - (V_t)\hat{i} \quad (1.20)$$

Rearranging Equation 1.20 into a more suitable form gives:

$$\bar{V}_{mt} = [(V_m \times \cos\theta) - V_t]\hat{i} + [V_m \times \sin\theta]\hat{j} \quad (1.21)$$

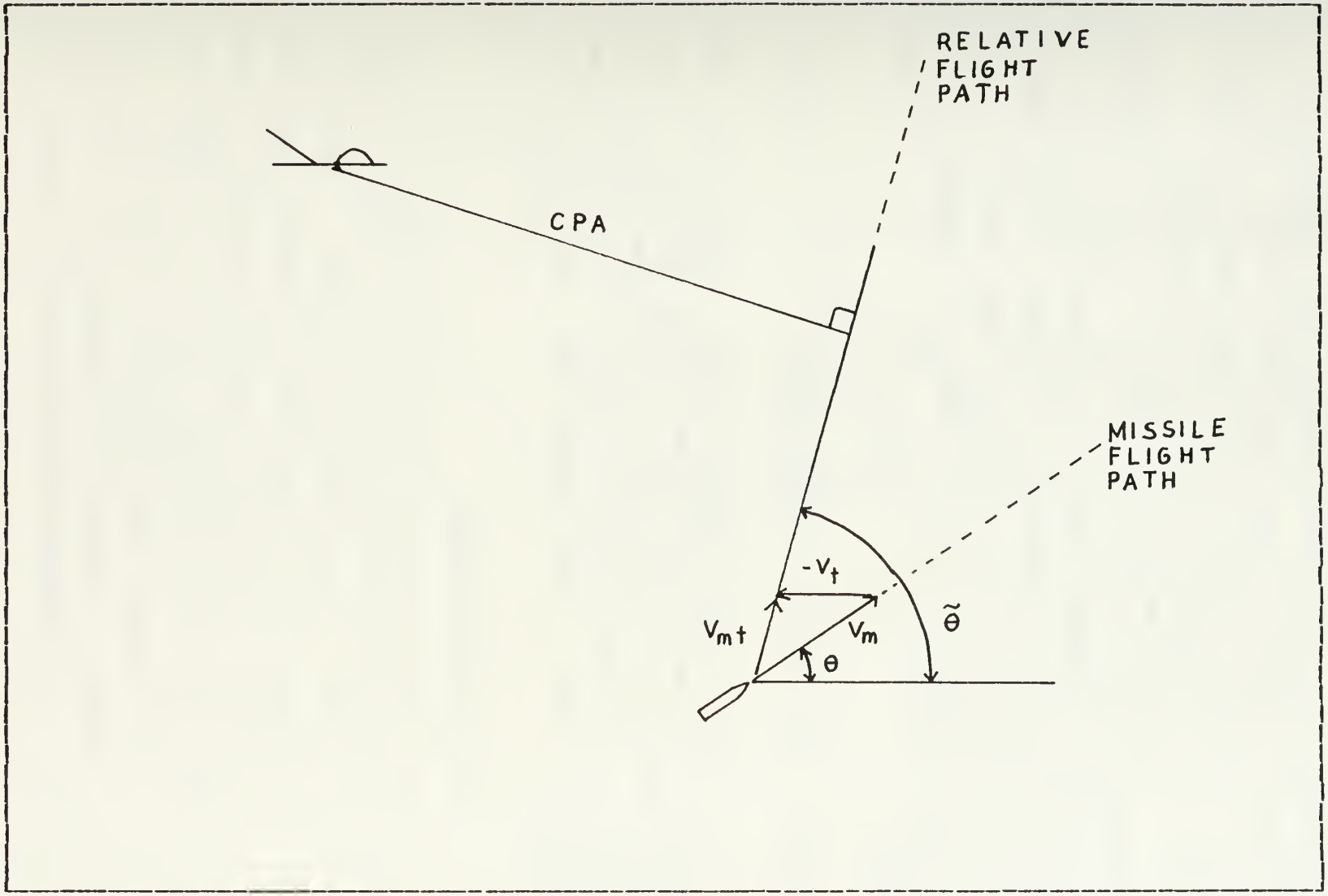


Figure 1.5 Encounter Scenario in the Local System

The derivation of the minimum missile miss distance in the local system follows the same procedure as for the global system, and since it leads to the same result as given by Equation 1.17, the derivation will not be repeated.

From the geometry of Figure 1.5, it is possible to determine if the missile will be a "late bird" or an "early bird". An "early bird" is where the closest point of approach (CPA) of the missile is in front of the target in the local system. A "late bird" is where the CPA of the missile is behind the target. This relationship may be determined graphically with the use of the following formula:

$$\tilde{\theta} = \text{Tan}^{-1} [(V_m \times \text{Sin}\theta) / [(V_m \times \text{Cos}\theta) - V_t]] \qquad (1.22)$$

Now that the missile miss distance has been calculated, and knowing the fragment spray density, a determination must be made as to whether or not the fragments hit the target, and if so, how many fragments strike the target. To accomplish this, the fragment miss distance must be determined.

4. Fragment Miss Distance

As was the case for missile miss distance, the fragment miss distance may be derived in either a global coordinate system or in a local coordinate system. Figure 1.6 depicts the fragment miss distance for the global coordinate system, and Figure 1.7 depicts the miss distance for the global system using vector notation. Figure 1.8 depicts the fragment miss distance in the local coordinate system. The derivation of the fragment miss distance in a global coordinate system follows. Referring to Figure 1.7, and using vector addition, the fragment miss distance, $\bar{c}$, is given by Equation 1.23.

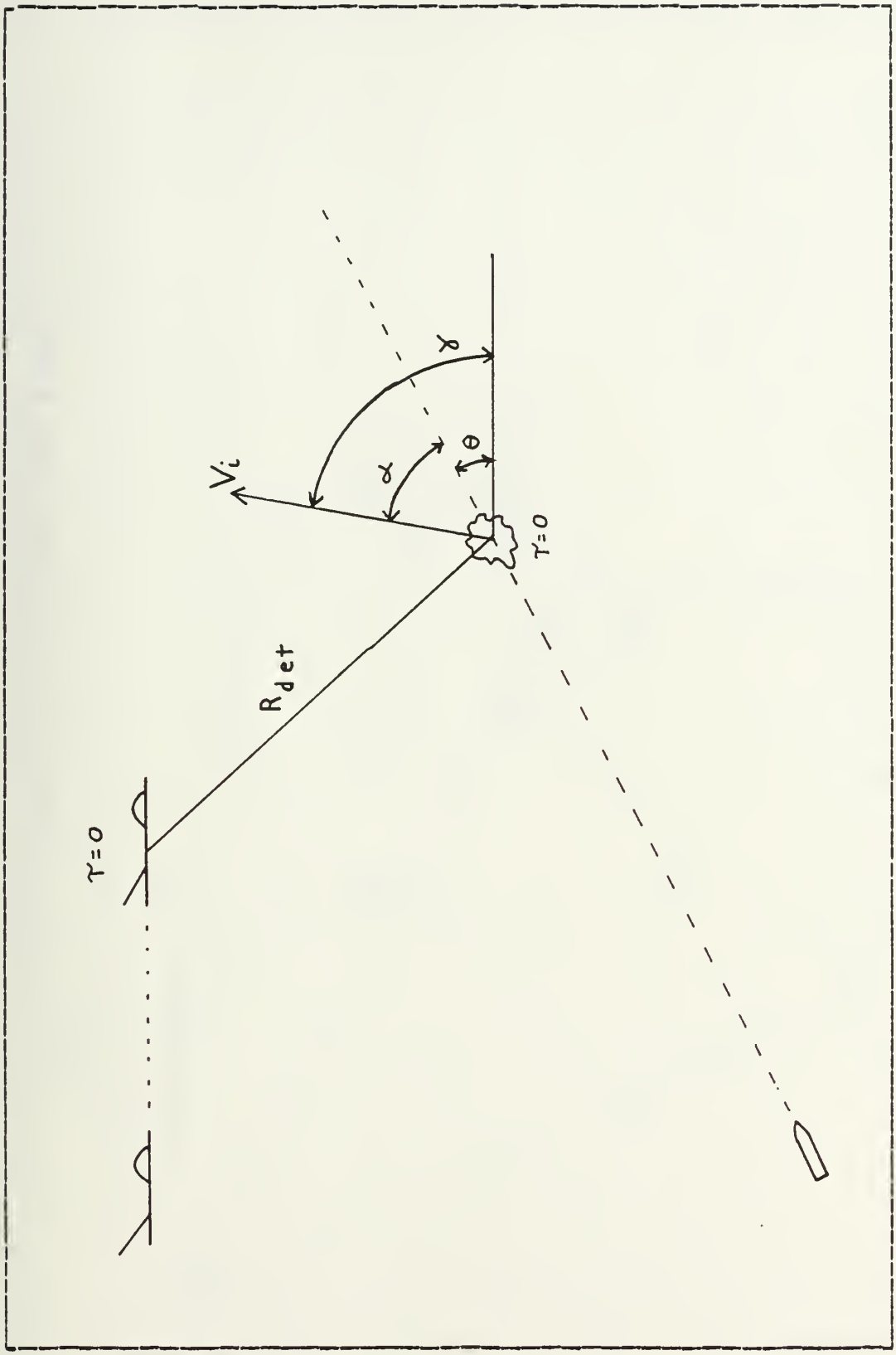


Figure 1.6 Fragment Miss Distance Depicted in Global Coordinate System

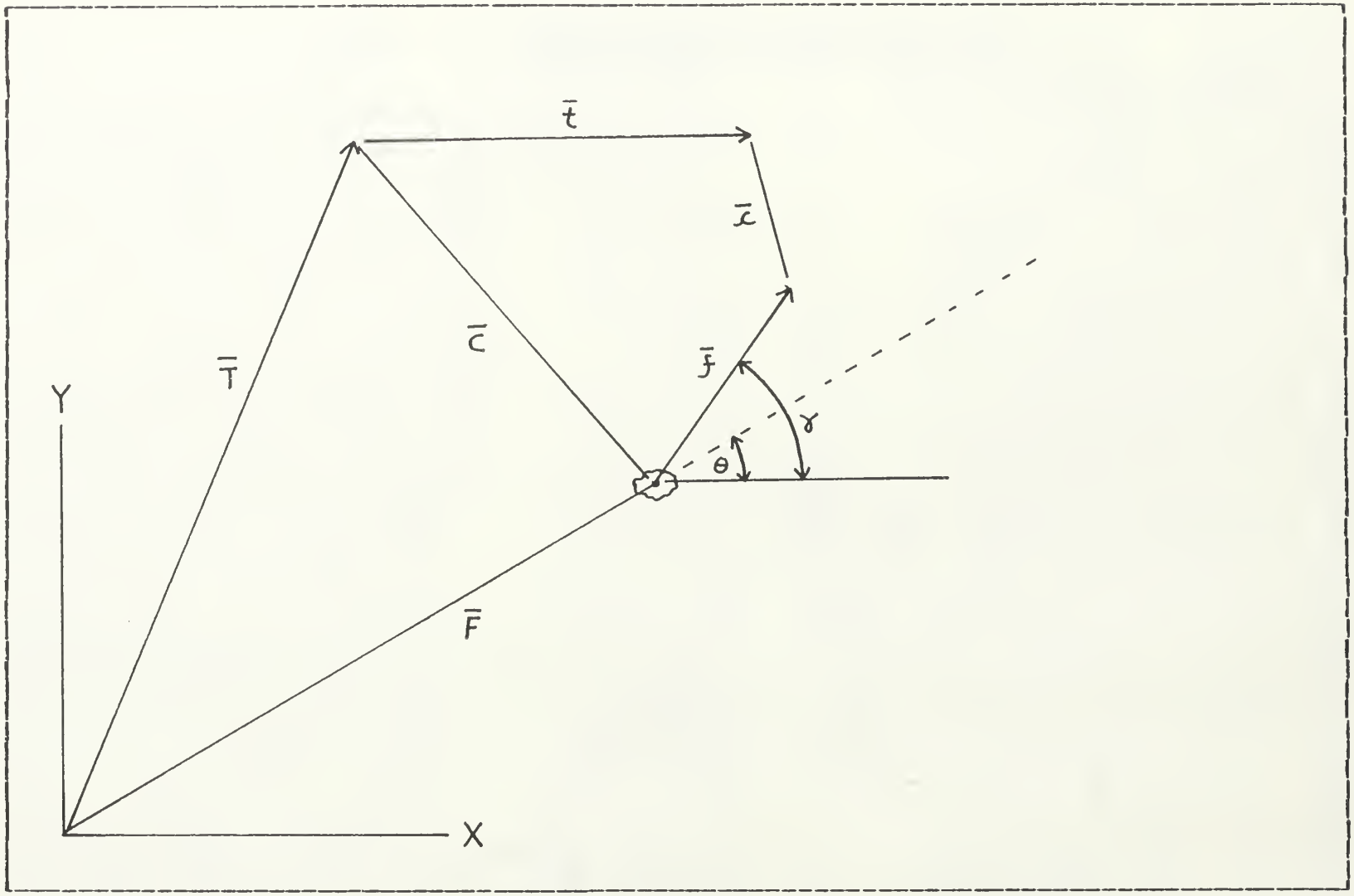


Figure 1.7 Fragment Miss Distance Using Vector Notation in Global Coordinate System

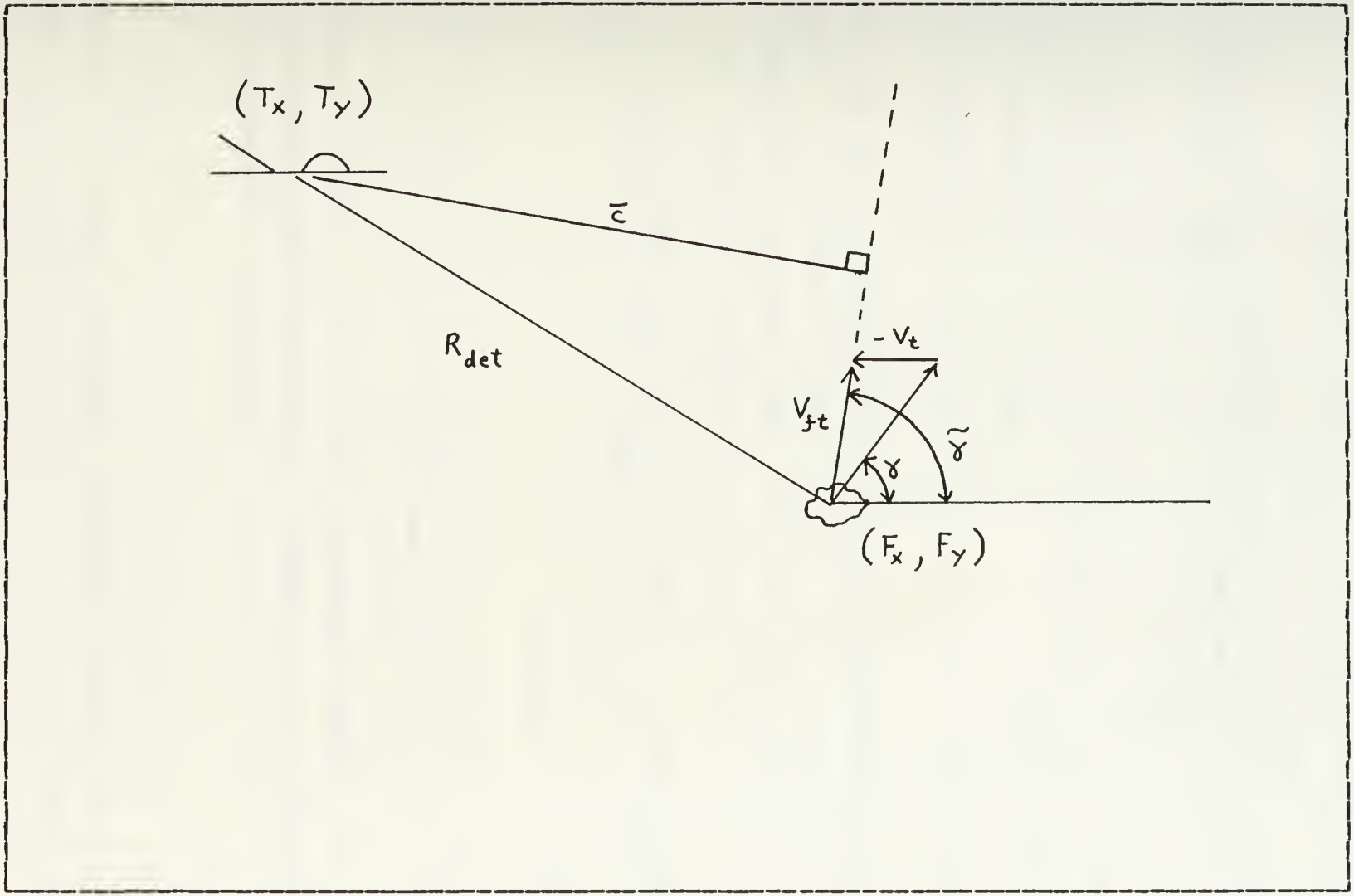


Figure 1.8 Fragment Miss Distance Depicted in Local Coordinate System

$$\bar{c} = (\bar{T} - \bar{F}) + (\bar{t} - \bar{f}) \tag{1.23}$$

The initial conditions are given by Equations 1.24 and 1.25. The conditions at some later time ($t = \tau$) are given by Equations 1.26 and 1.27.

$$\bar{T} = (T_x)\hat{i} + (T_y)\hat{j} \tag{1.24}$$

$$\bar{F} = (F_x)\hat{i} + (F_y)\hat{j} \tag{1.25}$$

$$\bar{t} = (V_t \times \tau)\hat{i} \tag{1.26}$$

$$\bar{f} = (V_i \times \tau \times \text{Cos}\gamma)\hat{i} + (V_i \times \tau \times \text{Sin}\gamma)\hat{j} \tag{1.27}$$

Substituting these conditions back into Equation 1.23, and solving for $\bar{c}$, leads to:

$$\bar{c} = [(T_x - F_x) - (\tau \times ((V_i \times \text{Cos}\gamma) - V_t))]\hat{i} \quad [(T_y - F_y) - (\tau \times V_i \times \text{Sin}\gamma)]\hat{j} \tag{1.28}$$

The magnitude of Equation 1.28 is:

$$|c| = [(T_x - F_x) - \{\tau \times ((V_i \times \text{Cos}\gamma) - V_t)\}]^2 + [(T_y - F_y) - (\tau \times V_i \times \text{Sin}\gamma)]^2 \tag{1.29}$$

To find the minimum fragment miss distance, τ for the minimum miss distance is needed. Taking the derivative of Equation 1.29 with respect to τ, and setting it equal to zero yields:

$$\tau = [\{(T_x - F_x) \times \{(V_i \times \text{Cos}\gamma) - V_t\}\} + \{(T_y - F_y) \times V_i \times \text{Sin}\gamma\}] / [((V_i \times \text{Cos}\gamma) - V_t)^2 + (V_i \times \text{Sin}\gamma)^2] \tag{1.30}$$

To find the minimum fragment miss distance, simply solve Equation 1.30 for τ, and substitute that value of τ into Equation 1.29. This derivation assumed that the fragment velocity was constant. If the fragment velocity is not constant, or reasonably so, then $\bar{f}$ is given by:

$$\bar{f} = \int_0^{\tau} [(V_i \times \cos\gamma)i + (V_i \times \sin\gamma)j] \, d\tau \qquad (1.31)$$

Solving Equation 1.31 gives:

$$\bar{f} = [(\cos\gamma)i + (\sin\gamma)j] \times \int_0^{\tau} Vi(\tau) \, d\tau \qquad (1.32)$$

Now that the fragment miss distance has been calculated, the number of fragments which strike the target will be determined.

5. Fragment Impacts on the Target

The number of hits, n, on the aircraft presented area at the aspect under consideration, A_p, is given by:

$$n = \rho \times A_p \qquad (1.33)$$

where ρ is the fragment spray density defined by Equation 1.8. Figure 1.9 depicts the fragment spray density striking an aircraft in the local coordinate system described earlier, where f_1 and f_2 represent the leading edge and trailing edge fragment vectors, respectively. The angle $\tilde{\gamma}$ is defined as:

$$\tilde{\gamma} = \tan^{-1} [(V_i \times \sin\gamma_i) / ((V_i \times \cos\gamma_i) - V_t)] \qquad (1.34)$$

Figure 1.10 depicts the same geometry in the global coordinate system described earlier. Recall from Equation 1.8 that the distance the fragments travel before striking the

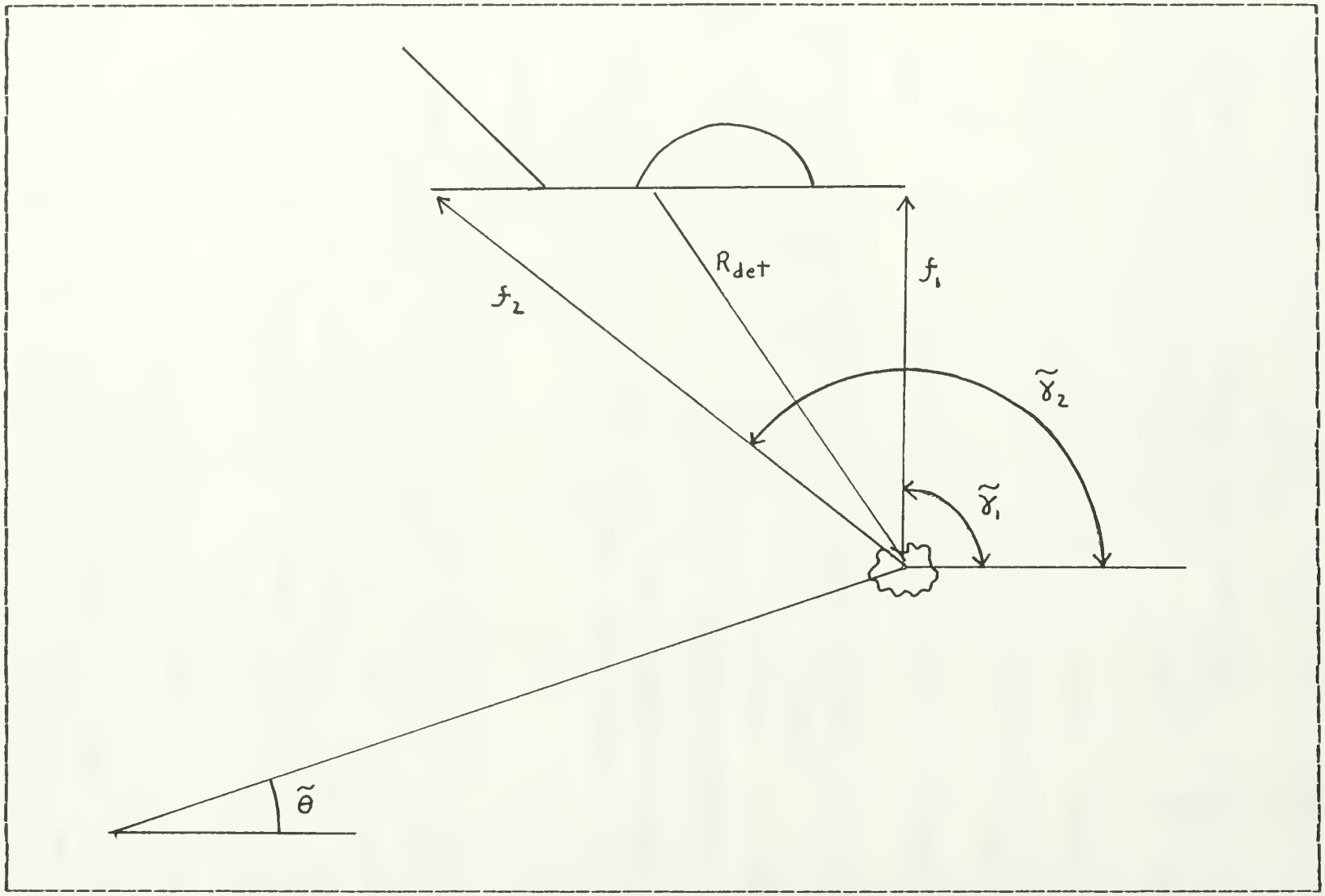


Figure 1.9 Fragment Impacts on Aircraft in Local System

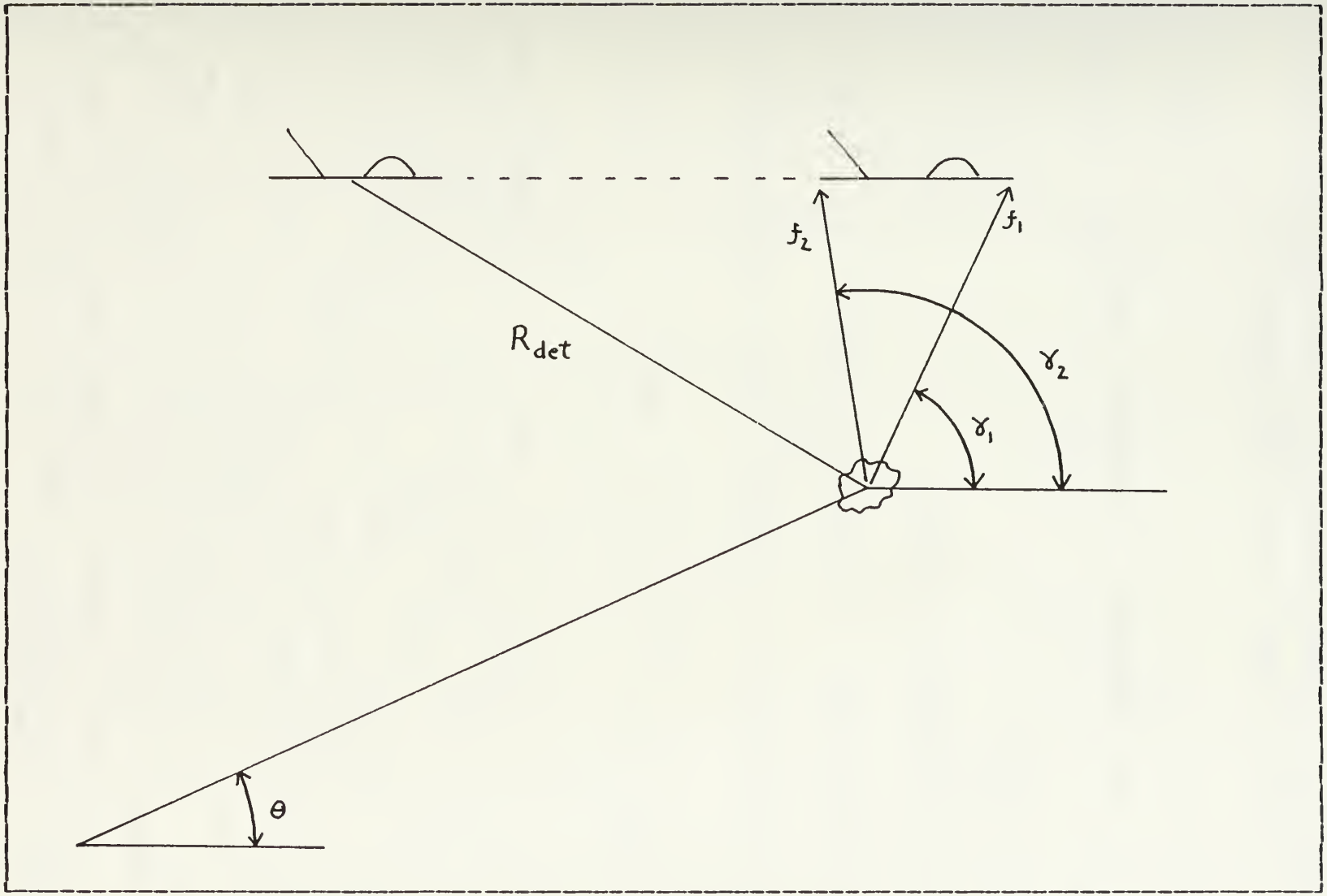


Figure 1.10 Fragment Impacts on Aircraft in Global System

aircraft, R, is needed and not R_{det}. The time from detonation to fragment impact in the global system is:

$$t = R / V_i \tag{1.35}$$

The time from detonation to fragment impact in the local system is:

$$t = R_{det} / V_{ft} \tag{1.36}$$

where V_{ft} is defined as follows:

$$V_{ft}^2 = [(V_i \times \cos\gamma) - V_t]^2 + [V_i \times \sin\gamma]^2 \tag{1.37}$$

Equating Equation 1.35 to Equation 1.36, and solving for R yields:

$$R^2 = R_{det}^2 / [1 - (2 \times (V_t / V_i) \times \cos\gamma) + (V_t / V_i)^2] \tag{1.38}$$

where V_t is the target velocity and V_i is the dynamic fragment velocity as defined by Equation 1.2. The extent of the fragment spray which strikes the aircraft, and the number and location of fragment hits, are dependent upon the encounter conditions. Figure 1.11 depicts the effects of varying the detonation distance to the target. In zone 1, the full fragment spray hits the target. In zone 2, all of the target is hit by part of the fragment spray. In zones 3 and 4, part of the fragment spray hits part of the target. In zone 1, with the target hit by the full fragment spray, the presented area of the aircraft may be determined as follows. Figure 1.12 depicts a frontal view of the fragment spray striking an aircraft. From Figure 1.12, the angle ζ is defined by Equation 1.39.

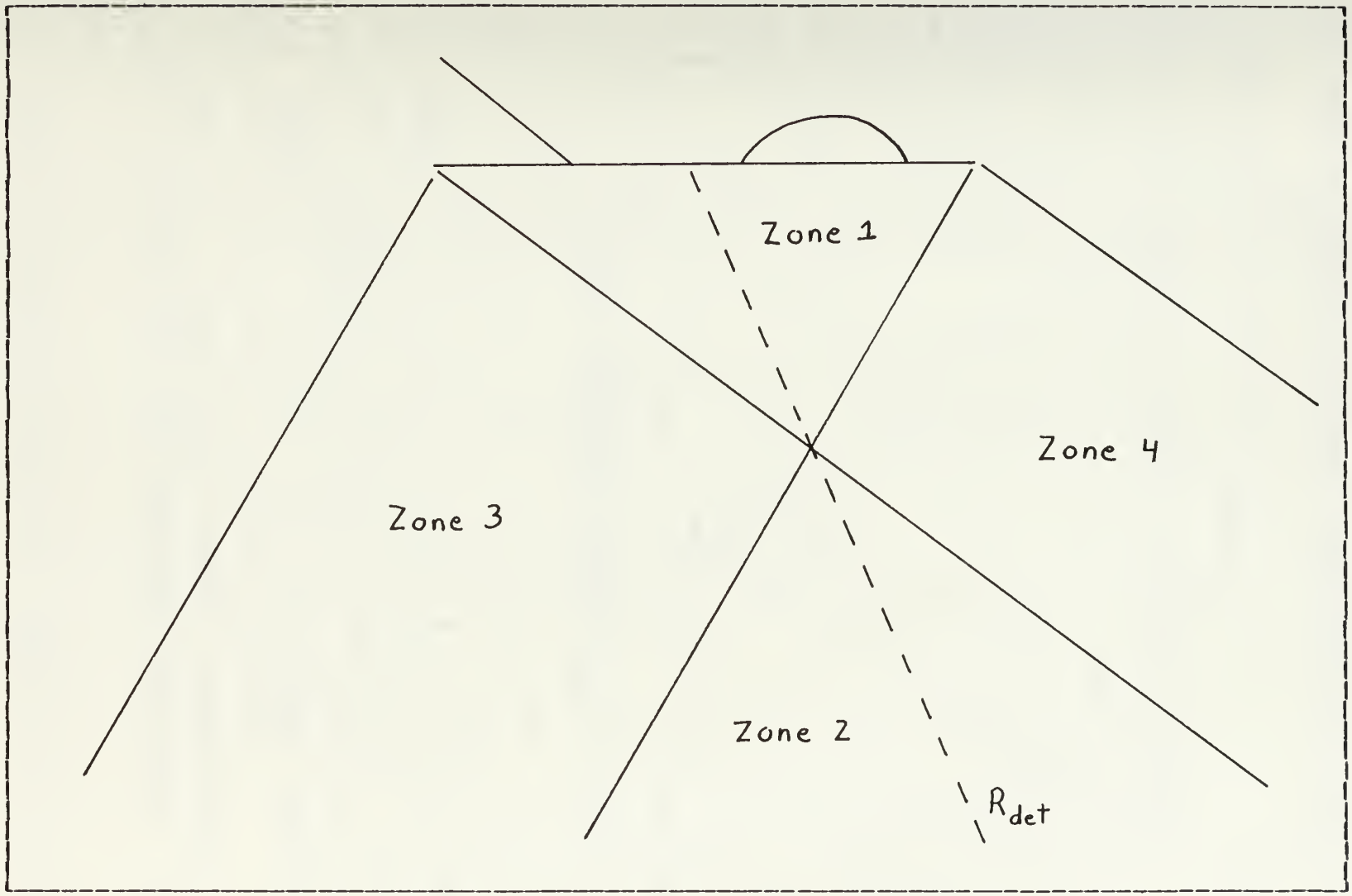


Figure 1.11 Effect of Varying Detonation Distance on Fragments Impacting on an Aircraft

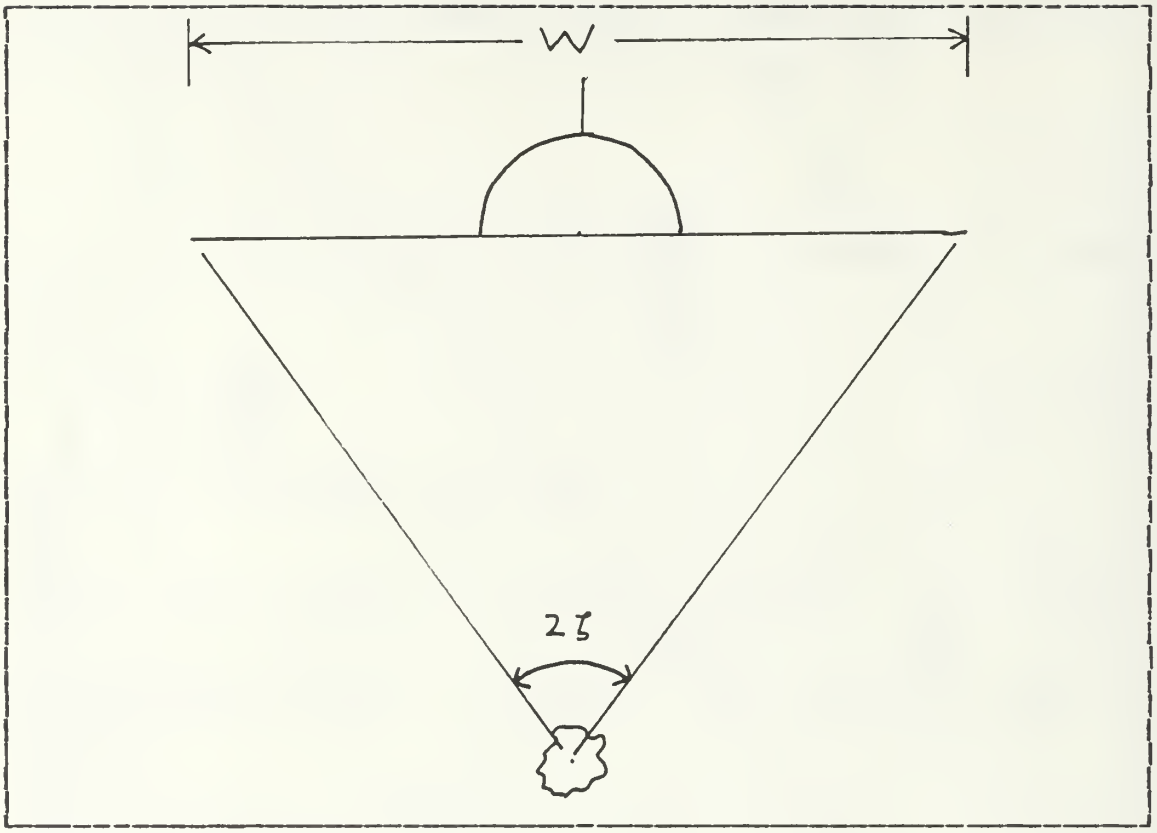


Figure 1.12 Frontal View of Fragments Impacting an Aircraft

$$\zeta = \text{Tan}^{-1} [W / (2 \times R \times \text{Sin}\,\phi)] \quad (1.39)$$

The circumferential length of the spray zone is defined as:

$$b = 2 \times \zeta \times R \times \text{Sin}\,\phi \quad (1.40)$$

where ϕ is the dynamic fragment spray angle of the center fragment. Substituting Equation 1.39 into Equation 1.40 yields:

$$b = 2 \times R \times \text{Sin}\,\phi \times \text{Tan}^{-1} [W / (2 \times R \times \text{Sin}\,\phi)] \quad (1.41)$$

The fragment spray zone covers a spherical area of:

$$\text{Area} = b \times R \times (\phi_2 - \phi_1) \tag{1.42}$$

Substituting Equation 1.42 into Equation 1.33 yields:

$$n = (N \times b) / [2 \times \pi \times R \times (\text{Cos}\,\phi_1 - \text{Cos}\,\phi_2)] \tag{1.43}$$

which determines the number of fragments which strike the aircraft in zone 1. The assumption has been made in this estimation that the fragment spray covers the entire presented area of the aircraft. If this is not the case, or if only a portion of the spray meridian hits a portion of the aircraft (zones 2, 3, and 4 in Figure 1.11), A_p, in Equation 1.33 must be reduced to the actual area that is struck by fragments. The extent of the fragment spray which does strike the aircraft, the number of fragment hits, the fragment approach directions, and where the hits occur are dependent upon the encounter conditions. For example, a detonation directly below the center of the aircraft in a head-on encounter will have a different result than from a detonation in the same place for a tail-chasing missile due to the difference in the relative closing velocity. Furthermore, changing the elevation angle of the missile at the time of detonation will change the results. With the number of fragments which strike the aircraft determined, the effects of fragments and penetrators striking an aircraft will be examined in the next chapter.

II. TARGET VULNERABILITY

A. INTRODUCTION

Target vulnerability refers to the inability of a target to withstand one or more hits by damage mechanisms (fragments, penetrators, incendiary particles, and blast) or a target's liability to serious damage or destruction when hit by enemy fire. Aircraft that are more vulnerable are softer, that is, they are more likely to be lost when hit. Therefore, aircraft vulnerability is essentially a measure of the toughness of an aircraft when all surviveability measures have failed and the threat interacts with the aircraft. From an air defense standpoint target vulnerability is good.

Each individual component of an aircraft has a certain level or amount of vulnerability. Each component's vulnerability then contributes, in some measure, to the overall vulnerability of the aircraft. The critical components of an aircraft are those components that are essential to the functioning of a system, and if the component performance is sufficiently degraded or if the component is rendered inoperative by combat damage, a target kill in some kill category will result. The systematic description, delineation, and quantification of the vulnerability of the individual components and vulnerability of the total aircraft is known as a vulnerability assessment.

B. IDENTIFICATION OF CRITICAL COMPONENTS

The first step in a vulnerability assessment is the identification of those components whose damage or loss could lead to an aircraft kill, and they are referred to as

critical components. This identification process is called critical component analysis. A component may be a critical component because it provides an essential function such as thrust, lift, or control. A component may also be a critical component because its mode of failure leads to the failure of a critical component that does provide an essential function. For example, a fuel tank in a wing can be perforated by a fragment, causing a slow fuel leak and eventual fuel depletion, with no substantial effect on the continued operation of the aircraft. In this situation, the wing fuel tank is not a critical component. On the other hand, the fragment impact and penetration of the wing tank could cause ignition of the fuel vapor in the ullage, with a subsequent fire or explosion and loss of the aircraft. In this case, the wing tank is definitely a critical component.

A general procedure has been developed for determining the critical components, their possible damage or failure modes, and the effects of the component damage or failure upon the continued operation of the aircraft. This procedure consists of: (1) a selection of the aircraft kill levels or categories to be considered, (2) an assembly of the technical and functional description of the aircraft, and (3) the determination of the critical components of the aircraft and their damage-caused failure modes for the selected kill levels.

1. Aircraft Kill Levels

To assess the vulnerability of both fixed wing and rotary aircraft in-flight, four kill categories have been defined. These kill categories are the Attrition Kill, the Mission (Mission Abort) Kill, the Forced Landing Kill, and the Mission Available Kill.

a. Attrition Kill

Attrition kill covers those aircraft with combat damage so extensive that it is neither reasonable nor economical to repair. The attrition category is divided into the six levels of kill listed below.

(1) KK Kill. This level of kill is associated with damage that will cause the aircraft to disintegrate immediately upon being hit. This kill level is also referred to as a Catastrophic Kill.

(2) K Kill. This level of kill is associated with damage that will cause an aircraft to fall out of manned control within 30 seconds after being hit.

(3) A Kill. This level of kill is associated with damage that will cause an aircraft to fall out of manned control within five minutes after being hit.

(4) B Kill. This level of kill is associated with damage that will cause an aircraft to fall out of manned control within 30 minutes after being hit.

(5) C Kill. This level of kill is associated with damage that will cause an aircraft to fall out of manned control before completeing its mission.

(6) E Kill. This level of kill is associated with damage that will cause an aircraft to sustain additional levels of damage upon landing and makes it uneconomical to repair as specified by the applicable Technical Orders, Technical Bulletins, and regulations.

b. Mission (Mission Abort) Kill

This category covers any aircraft with combat damage that prevents the aircraft from completing its mission. This is mission dependent and is divided into two levels; mission abort and mission kill. Mission abort covers aircraft which are not lost to inventory but cannot

complete their mission. Mission kill covers aircraft which fall out of manned control before completing their mission.

c. Forced Landing Kill

This category covers those aircraft with combat damage that forces the crew to execute a controlled landing (powered or unpowered). This category includes aircraft with damage which will require repairs for flight to another area and aircraft with damage which cannot be repaired on site but which can be recovered by a special team. This category has been restricted mainly to rotary wing aircraft which can land nearly anywhere either powered or by autorotation. It is more difficult for a damaged fixed wing aircraft to successfully execute a forced landing (and/or subsequent takeoff) since some prepared landing site is generally required.

d. Mission Available Kill

This category covers those aircraft that have landed with combat damage and will require repair before returning to mission ready status. There are different levels (intervals) for mission availability. The interval of time required to accomplish repairs is expressed in elapsed time, total man-hours, or combinations thereof.

2. Aircraft Description

At the beginning of any vulnerability study, as much as possible of the aircraft's technical and functional description must be gathered on each of the major systems of the aircraft. The aircraft's technical description consists of engineering data which documents the physical and functional relationships of the aircraft's subsystems. The types of physical descriptions utilized are general aircraft arrangement drawings such as three view and inboard

profiles, installation drawings, and schematic diagrams for the primary subsystems to include: airframe structure, propulsion system, fuel system, flight control system, pneumo-hydraulic system, aircrew, avionics system, and weapon and delivery systems. The suitability and quantity of data available to produce the necessary aircraft descriptions are functions of the status of the system within the acquisition or deployment phase. Aircraft technical descriptions should utilize all of the data base to include: engineering scaled drawings, subsystem functional descriptions, technical orders and manuals, and access to design personnel.

3. Critical Component Analysis

A critical component is any component that is essential to the functioning of a system, and if the component performance is sufficiently degraded or if the component is rendered inoperative by combat damage, a target kill in some kill category will result. For example, the engine in a single engine aircraft is a critical component for an A kill because its loss would lead to an aircraft loss within five minutes.

When two or more aircraft components are redundant, such as two engines, the loss of any one of the redundant components will not result in the loss of an essential function and hence, that component is not a critical component according to the definition given above. This assumes that the damage process and loss of one redundant component will not lead to the loss of any other redundant components. For example, if one engine of a twin engine aircraft starts to burn, the assumption is made that the fire will not spread to the other engine and destroy it. If this were to happen, there is no actual redundancy and both engines are nonredundant critical components. Since more than one hit can be

expected in a typical threat encounter, it is possible that all of the redundant components could be killed, leading to an aircraft kill. Therefore, the fact that a component is redundant does not eliminate it as a critical component. This requires that a distinction between the two kinds of critical components be made. In the past, nonredundant and redundant critical components have been referred to as singly vulnerable components and multiply vulnerable components, respectively. This terminology is confusing and will not be used here. A given component may be nonredundant with respect to a given kill category and redundant with respect to another kill category. For example, consider a twin engine helicopter. If the loss of either engine causes a mission abort, the engines are nonredundant for the mission abort category. If the loss of both engines is required to cause a crash or forced landing, the engines are redundant for these two kill categories.

The first step in a critical component analysis is to identify the flight and mission essential functions that the aircraft must perform in order to accomplish its mission. The second step is the identification of the major systems and subsystems that perform these essential functions. The third step is to conduct a Failure Mode and Effects Analysis (FMEA) to identify the relationships between each possible type of individual component or subsystem failure mode and the performance of the essential functions. The Fault Tree Analysis (FTA) is sometimes used to provide additional insight into the identification of critical components. The fourth step is to conduct a Damage Modes and Effects Analysis (DMEA). The DMEA relates component or subsystem failure modes to combat-caused damage. The combination of the third and fourth steps is referred to as the Failure Mode, Effects, and Criticality Analysis (FMECA). The last step in a critical component analysis is

a visual presentation of the list of critical components and/or a logical expression to identify the redundant and nonredundant critical components for the selected kill level. The visual presentation is referred to as a kill tree and the logical expression is referred to as a kill expression.

a. Flight and Mission Essential Functions

Flight essential functions are those system and subsystem functions required to enable an aircraft to sustain controlled flight. Mission essential functions are those system and subsystem functions required to enable an aircraft to perform its designated mission. The analysis should consider each phase of the mission. A typical mission for an attack aircraft would include such phases as takeoff, climb to cruise altitude, cruise to attack area, descent to attack altitude, target location, ordnance delivery, egress from the target area, climb to cruise altitude, return cruise, descent, and landing. The flight and mission essential functions should be identified and the priority for possible protection established for each of these phases. For example, the operation of the electronic weapons computer during takeoff is not a flight essential function, but it is a mission essential function during ordnance delivery. A particular level of operation should be identified for the flight essential functions such as lift, thrust, and control. For example, loss of one engine of a twin engine helicopter may not cause a total loss of lift and thrust, but it will lead to a reduction of performance capabilities. This loss of performance may not be acceptable in a hostile environment because the helicopter would become an easy target. Therefore, the continued operation of both engines may be required to prevent an attrition kill. Special functions must also be identified.

These include special functions such as those required for the vertical flight of a VTOL aircraft or those required for arrested landing aboard an aircraft carrier. A chart identifying some flight and mission essential functions and some of the mission phases for an attack helicopter is given in Figure 2.1.

b. System-Essential Functions Relationships

The ability of an aircraft to fly and to conduct its mission depends upon the continued operation of those systems and subsystems that perform the essential functions. If the aircraft is damaged in combat, the operation of certain subsystem components may be impaired or the component may cease to operate, and some essential functions may be lost. The severity and rapidity with which the loss of essential functions occur is directly related to the kill levels.

A general examination of each aircraft's systems and subsystems must be conducted to determine its specific contribution to the essential functions identified in the previous step. Figure 2.2 presents a sample tabulation of those systems and subsystems that contribute to the essential functions shown in Figure 2.1. A more detailed example of the relationship between the functions performed by one specific subsystem and the essential functions is shown in Figure 2.3.

c. Failure Mode and Effects Analysis (FMEA)

The Failure Mode and Effects Analysis is a procedure that: (1) identifies and documents all possible failure modes of a component or subsystem, and (2) determines the effects of each failure mode upon the capability of the system and/or subsystem to perform its essential functions. The FMEA process and requirements are defined in MIL-STD-785 and MIL-STD-1629A.

ITEM	ESSENTIAL FUNCTIONS	MISSION PHASES							
		Alert	Takeoff	Cruise to laager area	Cruise to holding position	Cruise to assault position	Engage targets	Return cruise	Land
	FLIGHT:								
1	Provide lift and thrust								
2	Provide controlled flight								
	MISSION:								
3	Communications • secured voice • unsecured voice • ICS								
4	Start systems								
5	Monitor systems								
6	Provide air data intelligence								
7	Maintain terrain clearance								
8	Employ IFF/ECM								
9	Navigate								
10	Locate/identify targets								
11	Employ weapons								

Figure 2.1 Some Essential Functions and Mission Phases for an Attack Helicopter

ITEM	ESSENTIAL FUNCTIONS	RELATED SYSTEMS									
		Electrical power	Hydraulic power	Flight control system	Propulsion system	Crew system	Fuel system	Power train/ rotor blade system	Avionics	Armament	Structure
	FLIGHT:										
1	Provide lift and thrust										
2	Provide controlled flight										
	MISSION:										
3	Communications										
4	Start systems										
5	Monitor systems										
6	Provide air data intelligence										
7	Maintain terrain clearance										
8	Employ IFF/ECM										
9	Navigate										
10	Locate/identify targets										
11	Employ weapons										

Figure 2.2 System-Essential Function Relationships

Subsystem function ID letter	SUBSYSTEMS / FUNCTIONS	ESSENTIAL FUNCTIONS										
		Provide lift and thrust	Provide controlled flight	Communications	Start systems	Monitor systems	Provide air data intelligence	Maintain terrain clearance	Employ IFF/ECM	Navigate	Locate/identify targets	Employ weapons
a	Generate electrical power											
b	Provide automatic control and protection of power generation											
c	Distribute electrical power											
d	Provide automatic protection of power distribution											
e	Provide power conversion (dc and low-voltage ac)											
f	Provide battery power											
g	Control subsystem loads											
h	Process and transmit subsystem data and power control signals											
i	Provide automatic electrical load management											
j	Provide controls and displays											
k	Provide illumination											

Figure 2.3 Subsystem Functions-Essential Function Relationships

The types of component failure modes generally considered in the FMEA include premature operation, failure to operate, failure during operation, failure to cease operation, and degraded or out-of-tolerance operation. A

determination of the major structural or aerodynamic damage tolerances is also performed during the FMEA. In addition, the effects of loss or major damage to aerodynamic surfaces on stability and control of the aircraft are required. Data generated should define the threshold for aerodynamic, structural, and control limits that can be tolerated for various flight conditions. A sample summary format for a FMEA for two flight control rod failure modes is given in Figure 2.4. Note in Figure 2.4 that the control rod is a critical component for an attrition kill when it jams, but not when is is severed.

The FMEA is applicable to both single component failures and multiple component failures. It is extremely important to consider multiple component failures, when the failure is due to combat damage, because of the likelihood that more than one component is damaged when the aircraft is hit.

The effects of a component failure should also include the consideration of any transients that might occur when the failure occurs. For example, consider a single engine, fly-by-wire, statically unstable aircraft with no mechanical flight controls as back-up. Suppose the engine-driven generator that supplies the electrical power to the flight control computer was to immediately cease operation and that the computer relies on an emergency ram-air turbine (RAT) for back up electrical power. The RAT is designed to be deployed into the airstream when the electrical power failure is sensed. However, this deployment takes time, during which the computer could be without sufficient power. This lack of power could cause problems with the fly-by-wire control system such as the loss of the SAS or the issuance of hardover commands to the control surfaces which could cause the aircraft to become uncontrollable, leading to an aircraft loss. Thus, the assumption of redundancy in the electrical power system is erroneous.

AIRCRAFT SUBSYSTEM	SUBSYSTEM		FAILURE MODE	EFFECT ON SUBSYSTEM	EFFECT OF DEGRADED SUBSYSTEM ON AIRCRAFT	AIRCRAFT KILL CATEGORY	SUPPORTING REFERENCES	COMMENTS
	COMPONENT	LOCATION						
FLIGHT CONTROLS	ROD 3127	LEFT WING	SEVER	AILERON GOES TO HARDOVER (UP) POSITION	HARDOVER EFFECT CAN BE BALANCED WITH OTHER CONTROL SURFACES	AIRCRAFT CAN FLY AND LAND USING OTHER CONTROL SURFACES	#1	1, 2
			JAM	PILOT'S CONTROL STICK IS LOCKED	NO CONTROL OF FLIGHT	ATTRITION	#7	

Figure 2.4 Sample FMEA Summary Format

d. Damage Mode and Effects Analysis (DMEA)

In the FMEA the cause of the component failure is not stipulated. The failure may or may not be related to combat damage. When specific component failures due to combat damage, such as mechanical damage to components caused by projectile or fragment penetration or damage caused by a fire or explosion, are identified and examined, the analysis is referred to as a DMEA. In the DMEA, the potential component or subsystem failures identified in the FMEA, as well as other possible damage-caused failures, are associated with the damage mechanisms and the damage processes. These failures are then evaluated to determine their relationship to the selected kill level. The quantification of the component kill criteria is also part of the DMEA, but this procedure is described in the vulnerability assessment presentation. The possibility of any secondary hazard that may be caused by the primary damage processes is also identified in the DMEA. Examples of secondary hazards are: ingestion of fuel by an engine, and seepage of toxic fumes from a fire into the cockpit. The DMEA is referred to as the criticality analysis of the FMECA.

The output of the DMEA can take many forms. The DMEA matrix is similar to the FMEA summary format shown in Figure 2.4 in which the components and their damage-caused failure modes are related to the kill level or category. Component redundancy relationships and the appropriate component kill criteria should also be indicated in the matrix. A sample DMEA matrix is given in Figure 2.5. A disablement diagram can add to the understanding of the DMEA matrix by graphically showing the locations of the components and stating the effects of component kills. A sample disablement diagram is presented in Figure 2.6.

AIRCRAFT _____

SYSTEM FLIGHT CONTROLS (MECHANICAL)

FMEA REF _____

COMPONENT NAME	COMPONENT NUMBER	DISABLEMENT DIAG. NO.	DAMAGE MODE	"KILL" CATEGORY						REMARKS	$P_{k/h}$ FUNC. NO.
				NON REDUNDANT			REDUNDANT				
				ATTRITION	MISSION ABORT	VERTICAL	ATTRITION	MISSION ABORT	VERTICAL		
STICK			BREAK OR DISABLE								
ASSEMBLY										DEGRADED	
(GRIP)	3001				X					FLIGHT CONTROL	32
		1									
			LOSS OF ELECTRICAL							LOSS OF CAS PITCH AND	
CAS SENSOR	3002		CONNECTIONS		X		X			ROLL CONTROL	32
		2	(LOSS OF CAS)							CONTROL THROUGH DEL.	
			LOSS OF ELECTRICAL							REVERSION TO MECH (IF DEL	
			AND MECHANICAL	X	X					IS LOST) (DEL - DIRECT	
			LINKAGES							ELECTRICAL LINK)	
RUDDER PEDALS	3006		BREAK OR DISABLE								
ARMS	3007		ONE ARM								32
SUPPORT	3008	3									32
FEEL SPRING SUPPORT	3301		BREAK OR DISABLE								32
SPRING	3302		SUPPORT, FEEL							NO ELECTRICAL	32
TRANSDUCER	3303		SPRING ASSY, OR	X	X					INPUTS TO	24
			TRANSDUCER							RUDDERS	

Figure 2.5 A Sample DMEA Matrix

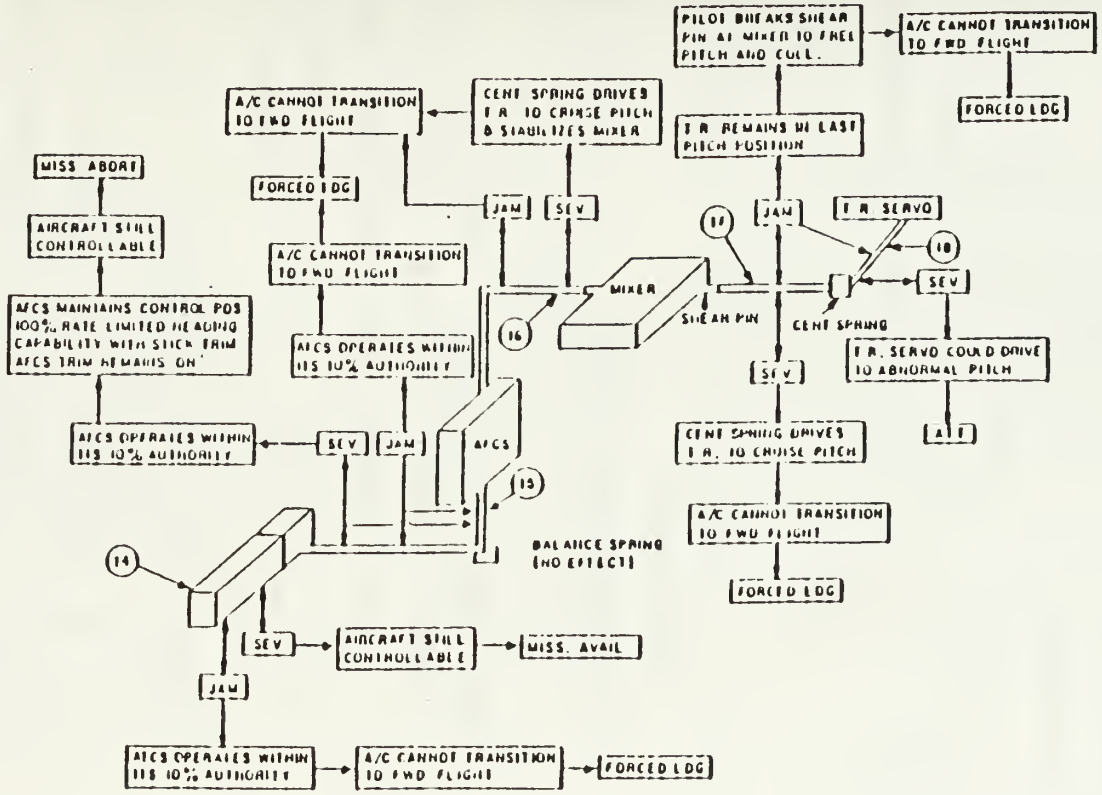

PILOT BREAKS SHEAR PIN AT MIXER TO FREE PITCH AND COLL.
A/C CANNOT TRANSITION TO FWD FLIGHT
FORCED LDG
T R REMAINS IN LAST PITCH POSITION
CENT SPRING DRIVES T R TO CRUISE PITCH & STABILIZES MIXER
A/C CANNOT TRANSITION TO FWD FLIGHT
FORCED LDG
MISS ABORT
AIRCRAFT STILL CONTROLLABLE
AFCS MAINTAINS CONTROL POS 100% RATE LIMITED HEADING CAPABILITY WITH STICK TRIM AFCS TRIM REMAINS ON
A/C CANNOT TRANSITION TO FWD FLIGHT
AFCS OPERATES WITHIN ITS 10% AUTHORITY
JAM
SEV
MIXER
SHEAR PIN
CENT SPRING
T R SERVO
SEV
T R. SERVO COULD DRIVE TO ABNORMAL PITCH
A F
AFCS OPERATES WITHIN ITS 10% AUTHORITY
SEV
JAM
AFCS
CENT SPRING DRIVES T R. TO CRUISE PITCH
A/C CANNOT TRANSITION TO FWD FLIGHT
FORCED LDG
BALANCE SPRING (NO EFFECT)
SEV
AIRCRAFT STILL CONTROLLABLE
MISS. AVAIL
JAM
AFCS OPERATES WITHIN ITS 10% AUTHORITY
A/C CANNOT TRANSITION TO FWD FLIGHT
FORCED LDG

Figure 2.6 A Sample Disablement Diagram

There are many different kinds of damage-caused failures or kill modes that can occur within each of the systems of an aircraft. Failure modes for an aircraft are the various ways in which the aircraft can fail to be maintained in the required mode of flight or fail to perform its mission. These failure modes are constituted by the loss of, or serious degradation of, structural integrity, power, flight control or mission required equipment, or lift. Failure modes are established for a given aircraft and mission with respect to preestablished minimum requirements for performance of the aircraft and are related to the critical components of the aircraft. Some of the most important ones are listed in Table 1 and described below. The order of the systems listed is indicative of their contribution to the total aircraft vulnerability.

(1) Fuel System Kill Modes. The following is a listing and brief description of the potential fuel system kill modes.

Fuel Supply Depletion. This kill mode is caused either by damage to fuel storage components that results in excessive leakage leading to a significant reduction in the amount of fuel available for aircraft operation, or by damage to fuel pumping and transfer systems that prevents fuel from reaching the engine(s).

In-Tank Fire and Explosion. Fire and explosion can be caused by the ignition of the fuel-air mixture in the ullage by incendiary particles or by a hot tank wall. The in-tank fire or explosion can cause substantial damage to the tankage and adjacent structure and components, and the fire may quickly spread to other parts of the aircraft.

Void Space Fire and Explosion. This can be caused by fuel leakage into void spaces or dry bays (adjacent to punctured fuel tanks and lines) that is

TABLE 1

System Damage-Caused Failure Modes

Fuel

- Fuel Supply Depletion
- In-Tank Fire/Explosion
- Void Space Fire/Explosion
- Sustained Exterior Fire
- Hydraulic Ram

Power Train and Rotor Blade/Propellor

- Loss of Lubrication
- Mechanical/Structural Damage

Electrical Power

- Severing or Grounding
- Mechanical Failure
- Overheating

Propulsion

- Fuel Ingestion
- Foreign Object Ingestion
- Inlet Flow Distortion
- Lubrication Starvation
- Compressor Case Perforation or Distortion
- Combustor Case Perforation
- Turbine Section Failure
- Exhaust Duct Failure
- Engine Control and Accessories Failure

Crew

- Injury, Incapacitation or Death

Armament

- Fire/Explosion

Flight Control

- Disruption of Signal Path
- Loss of Control Power
- Loss of Aircraft Motion Data
- Damage to Control Surfaces and Hinges
- Hydraulic Fluid Fire

Structural

- Structure Removal
- Pressure Overload
- Thermal Weakening
- Penetration

Avionics

- Penetrator/Fragment Damage
- Fire/Explosion
- Radiation Damage

subsequently ignited by incendiary particles, by hot metal surfaces, or by the hot gases from punctured bleed air lines or engine cases. Fire or explosion in the enclosed spaces can eventually cause significant damage to nearby subsystem components and structure that would result in their failure. The generation of smoke and toxic fumes may also occur and migrate to crew stations, causing a possible mission abort, forced landing, or aircraft abandonment.

Sustained Exterior Fuel Fire. This kill mode is caused by damage to fuel tank walls resulting in fuel spillage onto the exterior of the aircraft which is subsequently ignited, producing a sustained fire. Sometimes the exterior fire is snuffed out by the airflow over the surface; however, the condition of the damaged surface, the altitude, and the flight speed may prevent this from occuring.

Hydraulic Ram. Damage to container walls or components within the container caused by the intense pressure waves generated in the contained liquid by penetrators or fragments passing through the liquid is referred to as hydraulic ram damage. The fluid pressure can cause large cracks and gaping holes in the container walls, leading to excessive leakage either externally or internally into dry bays, engine inlets, etc..

(2) Propulsion System Kill Modes. The following kill modes of the propulsion system have been observed.

Fuel Ingestion. Fuel ingestion is caused by fuel entering the engine air inlet following rupture of walls that are common to both a fuel tank and the inlet. Fuel ingestion effects normally include compressor surge, severe stall, unstable burning in the tail pipe, and/or engine flameout.

Foreign Object Ingestion. Foreign objects consist of projectiles, fragments, and pieces of damaged aircraft components which enter the engine inlet and subsequently damage the fan and compressor blades. This could cause an engine failure or the throwing of blades through the engine case, leading to additional component damage.

Inlet Flow Distortion. Distortion of air flow to the engine can be so severe as a result of combat damage to the inlet that uncontrollable engine surging or engine failure occurs.

Lubrication Starvation. Penetrator, fragment, or fire damage to the lubrication circulation and cooling subsystem can result in loss of lubrication and subsequent deterioration of bearing surfaces, followed by engine inoperability. Loss of lubrication failures are most often related to the bearings, where loss of heat removal eventually results in bearing seizure.

Compressor Case Perforation or Distortion. This kill mode is caused by penetrator or fragment penetration through the case, by distortion of the case, or by damaged compressor blades exiting through the case.

Combustor Case Perforation. Penetrator or fragment penetration and holing of the combustor case, with subsequent hot gas emission or torching through the hole, can cause secondary damage effects, such as severe heating of adjacent fuel tanks or control rods, and can also cause a combustion pressure drop that may result in a significant loss of engine power.

Turbine Section Failure. Turbine failure can be caused by penetrator or fragment damage to the turbine wheels, blades, and case. This results in a loss of engine power or secondary perforation and possible fire damage.

Exhaust Duct Failure. Penetration by penetrators and fragments into the exhaust duct may result in damage to nozzle control lines and actuator mechanisms and possible fuel spillage and secondary fire if an augmentor is operating at the time of the hit.

Engine Controls and Accessories Failure. A kill of the controls and accessories can be caused by penetrator, fragment, or fire damage. The result can be loss of control of the engine or loss of one of the important accessories.

(3) Flight Control System Kill Modes. Some possible flight control kill modes are listed below.

Disruption of Control Signal Path. Severence or jamming of the mechanical or electrical path that transmits the control signals from the pilot to the control surfaces or the actuators can partially or totally incapacitate the control system.

Loss of Control Power. Control power can be lost as a result of damage to hydraulic power components which causes a loss of hydraulic pressure. Types of power system damage are thermal degradation due to fire, perforation of hydraulic reservoirs, cylinders, or lines leading to a loss of hydraulic fluid, and deformation of hydraulic components, actuators, or lines that cause a hydraulic lock or jammed condition.

Loss of Aircraft Motion Data. Damage to the aircraft motion sensors or to the sensor data signal paths to the flight control computer can prevent the autopilot and the stability augmentation system from properly controlling the motion of the aircraft. The results can vary from a partial loss of control, leading to a mission abort, to the loss of an out-of-control aircraft. These components are relatively soft and are easily damaged or severed by penetrators, fragments, and fire.

Damage to Control Surfaces and Hinges. Penetrators, fragments, blast, and fire damage can result in the physical removal of a portion or all of a flight control surface or in the jamming of the hinges, rods, and other linkages between the servoactuators and the control surfaces.

Hydraulic Fluid Fire. Fires can result from the ignition of pressurized or gravity-leaked hydraulic fluid, and smoke or toxic fumes from the fire can affect the crew.

(4) Power Train and Rotor Blade/Propeller System Kill Modes. Some of the possible kill modes within the power train and rotor blade system of helicopters and propeller driven fixed-wing aircraft are described below.

Loss of Lubrication. This kill mode can occur due to projectile or fragment perforation of oil or grease containing components, with subsequent loss of lubrication oil or grease. Lubrication starvation is especially critical in oil-cooled helicopter transmissions, where the oil systems are not self-contained and usually consist of externally mounted components, such as sumps, filters, coolers, and interconnecting lines and hoses. Loss of lubrication prevents the removal of heat and lubrication of rubbing surfaces, which eventually results in component seizure. In helicopter transmissions and gear boxes, failures are often catastrophic, causing case rupture and fire after input pinion failures and rotor blade seizure after planetary assembly failures.

Mechanical/Structural Damage. Mechanical or structural failure of power train components can be caused by fragment and penetrator impact or penetration, or by fire. Bearings, gears, and shafts are prone to damage and failure when hit, shafts can be severed, and bearings

and gears can jam. Chips and debris from damaged components or structure can jam the oil pump, causing loss of lubrication. Rotor blades and propellers when hit can result in rotor unbalance, blade instability, blade out-of-track, and loss of lift. Rotor unbalance is perhaps the most critical consequence of ballistic damage and occurs when a portion of the blade is removed. This loss of mass in one blade can cause large, alternating hub forces and intense cockpit and control vibrations, leading to structural failure or loss of control. Blade instability is caused by a reduction of blade stiffness due to damage and can result in severe flutter or divergent pitch oscillation that can be catastrophic. Blade out-of-track is usually a less severe result of the reduction of blade stiffness, but it could result in blade contact with the fuselage. Although some loss of lift normally accompanies any ballistic damage, the consequences are usually not as catastrophic as those associated with the other types of blade reactions.

(5) Crew System Kill Modes. The inability of the pilot and his or her replacement to operate the aircraft because of injury, incapacitation, or death will usually lead to an aircraft kill in a very short period of time.

(6) Structural System Kill Modes. The structural system is usually the toughest system on the aircraft. However, structural damage can be sufficient to cause an aircraft kill. Some possible structural kill modes are listed below.

Structure Removal. Physical severence or complete loss of large portions of the load-carrying aircraft structure caused by multiple penetrators and fragments, blast, fire, or radiation effects can result in either an immediate or a delayed aircraft loss.

Pressure Overload. Immediate failure or subsequent failure under maneuver loads can be caused by

external blast effects which result in over-stressing the load carrying structure.

Thermal Weakening. Structural failure can occur to portions of the load-carrying structure as a result of internal void space fires, externally sustained fires, or radiation over a portion of the aircraft surface.

Penetration. A single penetration of one load-carrying member will usually not cause structural failure; several members must be penetrated or cut before failure can occur. Since the likelihood of structural failure from penetration by a few fragments or armor-piercing projectiles is extremely small, this type of failure would most likely result from continuous rod warhead effects.

(7) Electrical Power System Kill Modes. The failure of electrical system components is due to the severing or grounding of electrical circuits, the destruction or unbalancing of rotating components, such as generators and alternators, and the penetration or overheating of batteries.

(8) Armament System Kill Modes. Two major reactions can occur when gun ammunition, bombs, rockets, and missiles are hit by a damage mechanism. One is a sustained fire in the magazine that could cause cook-off or detonation of the stored ammunition, and the other is a severe explosion of either the armament or the propellant.

(9) Avionics System Kill Modes. Avionics components are usually very soft and are easily damaged by penetrators and fragments, blast, radiation, and thermal hazards, such as fire or hot gas torching. Their kill mode is usually failure to operate, although a degraded operation is possible.

e. Fault Tree Analysis (FTA)

As described in the preceding section, the FMEA is a bottom-up approach to determine an aircraft's critical components. In the FMEA, the failure of a component is assumed and the consequences of that failure are identified. Another procedure for identifying critical components is the Fault Tree Analysis. The FTA is a top-down approach which starts with an undesired event and then determines what event or combination of events can cause the undesired event. The Fault Tree Analysis is one of the principal methods of system safety analysis, and can include both hardware failures and human effects. The generic fault tree diagram shown in Figure 2.7 demonstrates the logic symbology used in the Fault Tree Analysis.

The undesired event U can only occur when event A and event B occurs. (This is the logical AND gate). Event A can occur when event C or event D occurs, or if both event C and event D occurs. (This is the inclusive OR gate). Event B can occur when event E or event F occurs, but not when both event E and event F occurs. (This is the exclusive OR gate). Because the undesired events of interest here are failures caused by damage, the FTA will be referred to as the Damage Tree Analysis.

A portion of a damage tree diagram for a twin-engine aircraft with a single fuel supply source to both engines is illustrated in Figure 2.8c. The undesired event is an aircraft attrition kill. An attrition kill occurs if the aircraft can neither fly nor land. The aircraft cannot fly if it loses lift, or thrust, or control. Loss of thrust will occur when both engines fail or when the common fuel supply to both engines fails. Leakage from damage caused by penetration or hydraulic ram from the feed tank that supplies fuel to both engines will cause the fuel system to

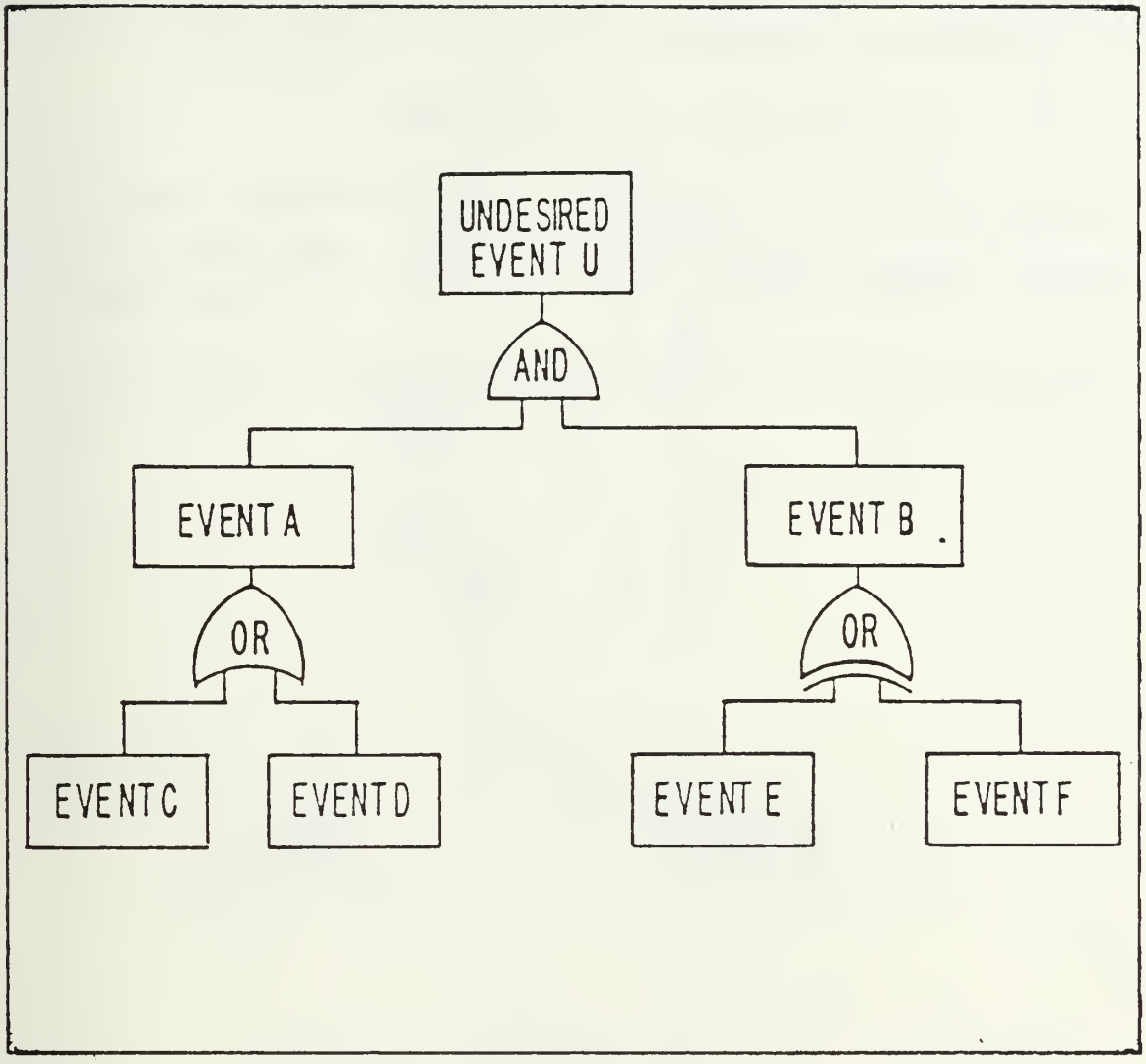


Figure 2.7 A Generic Fault Tree Diagram

fail. The left engine can fail due to engine damage or the loss of the left engine fuel supply. The left engine fuel supply system can fail due to penetration of many of the fuel transfer components from the feed tanks to the engine combustor, or these components can fail due to fire caused by leaking fuel, leaking hydraulic fluid, or a holed combustor. The left engine can fail due to damage caused by fuel ingestion, penetration of the engine compressor, combustor,

or turbine, loss of lubrication, fire caused by leaking fuel, or damage to the engine controls or accessories.

f. Kill Trees and Kill Expressions

Results of the steps described above leads to the identification of a set of critical components in a particular aircraft design, for a specific operational mode

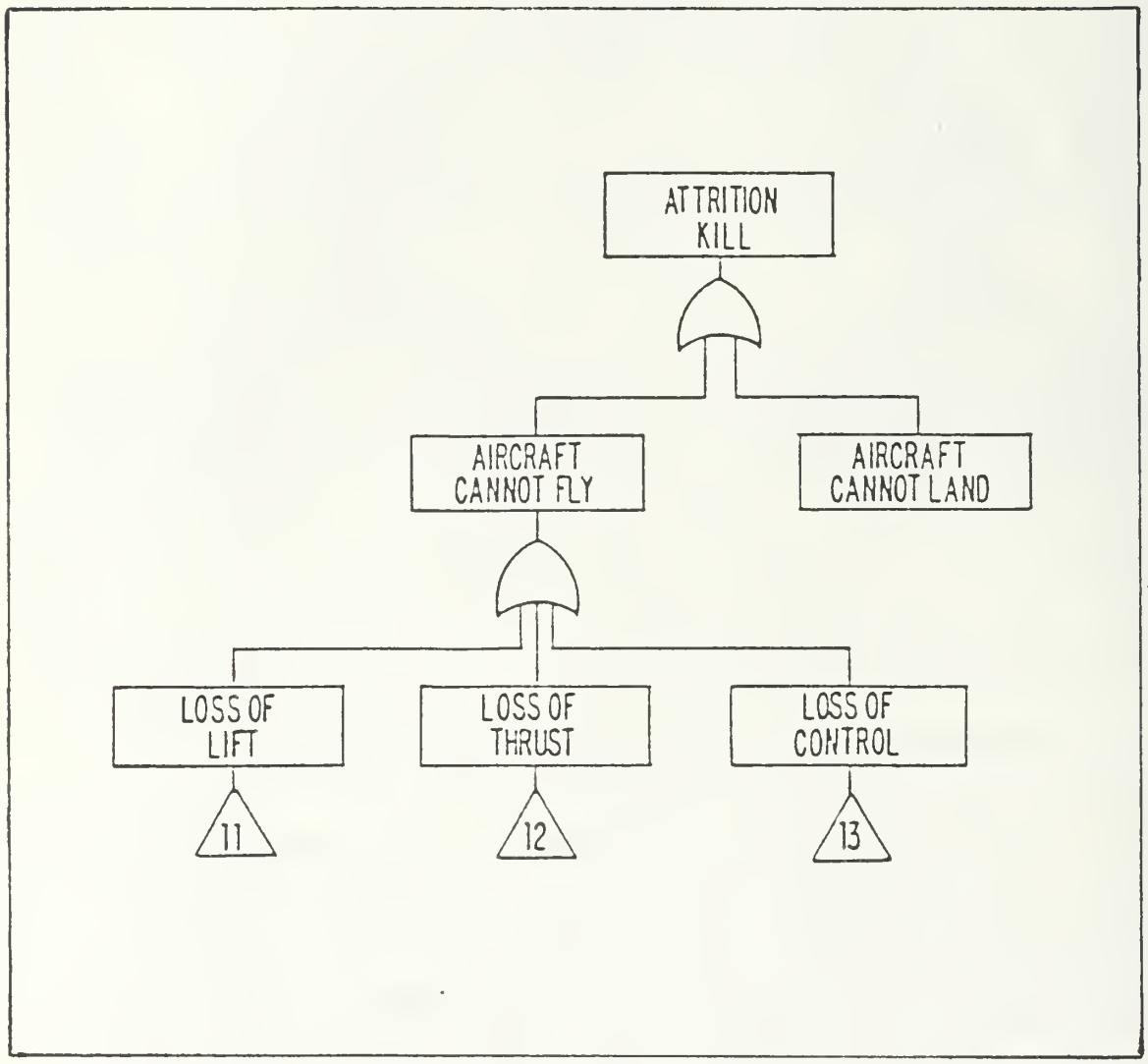


Figure 2.8a Portion of a Damage Tree Diagram for a Twin Engine Aircraft

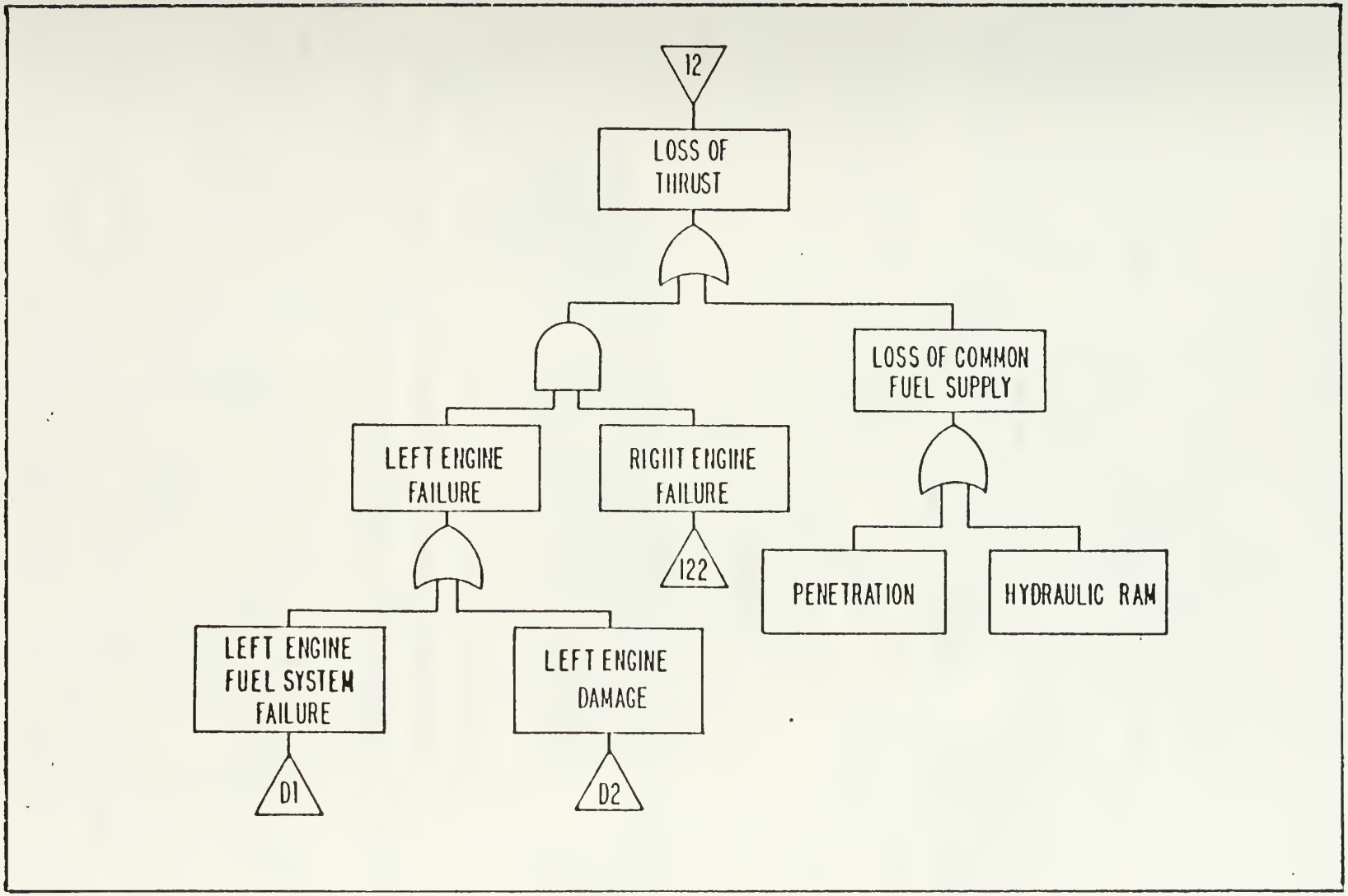


Figure 2.8b Portion of a Damage Tree Diagram Cont'd

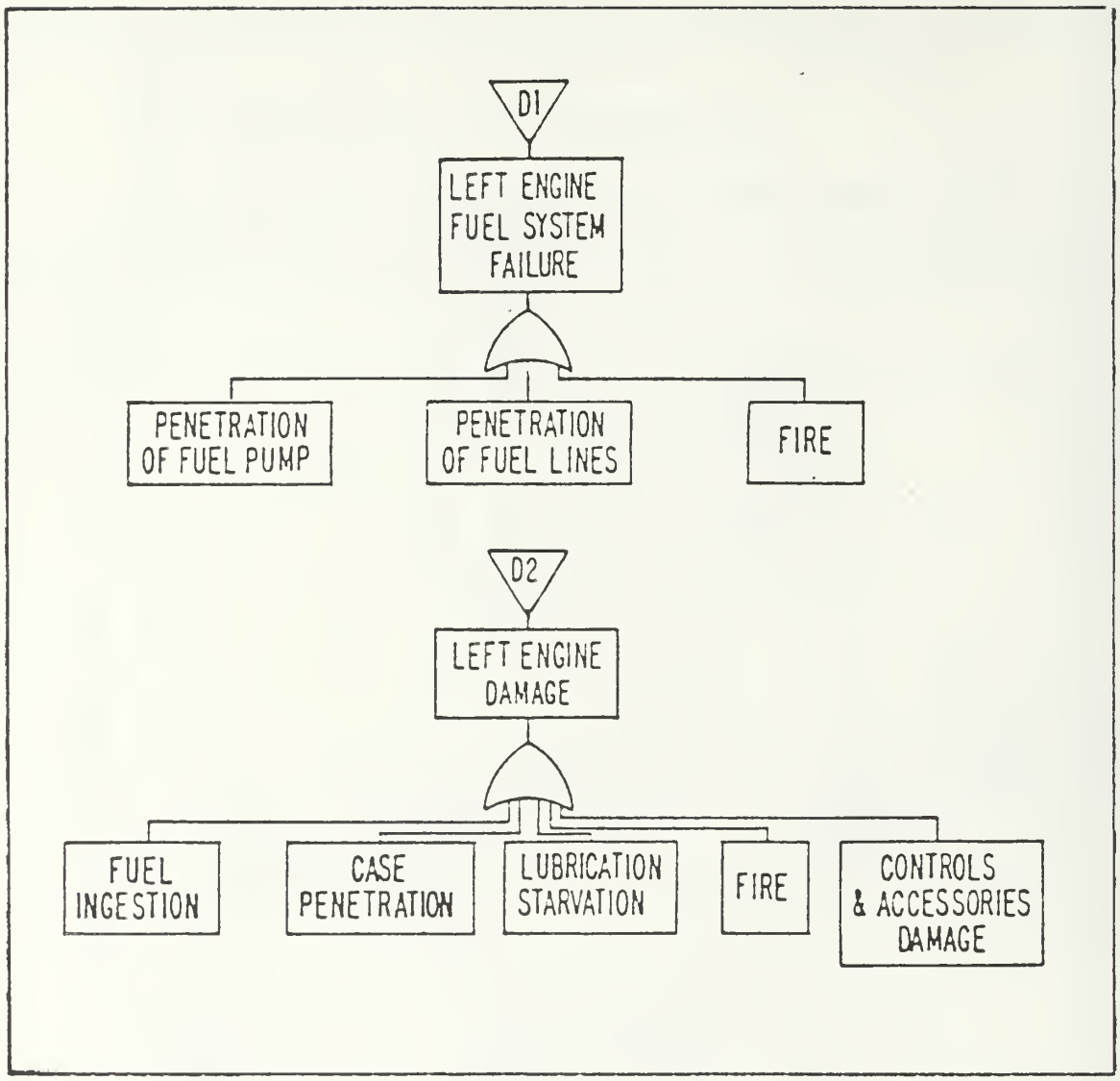


Figure 2.8c Portion of a Damage Tree Diagram Cont'd

and selected kill level. Each critical component either makes a singular contribution to an essential function or each component is one of two or more redundant components, each of which can make the necessary contribution. The distinction between nonredundant critical components and redundant critical components is extremely important and will be demonstrated in the following sections.

(1) Typical Critical Components. For a two engine, single pilot helicopter, the following nonredundant components are potential critical components for an attrition kill: (1) Flight control system components (rods, rod ends, bellcranks, pitch links, swashplate, hydraulic actuators, collective lever, and control pedals), (2) Rotor blade and power train components (blades, drive shafts, rotor heads, main transmission, and gear boxes), (3) Fuel system components (fuel cells, the sump, lines, and valves), (4) Pilot, and (5) Tail boom.

The following redundant components are potential critical components for an attrition kill: (1) Propulsion system components (engine and engine mounts), (2) Hydraulic subsystem components, and (3) Structural elements.

For a single engine, single pilot, fixed-wing aircraft, some potential redundant and nonredundant critical components are: (1) Pilot, (2) Flight controls in the cockpit and the pitch axis flight control components, (3) Hydraulic reservoirs, high-pressure lines, components and actuators, (4) All fuel tanks, components, lines, and shut-off valves, (5) Engine fan, compressor, turbine, and combustor sections, drive shaft and bearings, engine mounts, and the lubrication and fuel supply components, (6) Major structure, such as wing box spars, fuselage longerons, and the horizontal and vertical stabilizer spars and attachments, (7) External ordnance and the ammunition storage drum, (8) Liquid oxygen (LOX) bottle and components, and (9) Liquid-cooled avionics with a flammable coolant.

(2) The Kill Tree. A visual illustration of the critical components and of the contribution of component redundancy is provided by the kill tree. In order to kill the aircraft a complete cut through the tree trunk is required. A sample kill tree for a two engine, two pilot

helicopter is shown in Figure 2.9. For example, according to the kill tree in Figure 2.9, a loss of the pilot and either the co-pilot or the co-pilot's controls will lead to an aircraft kill, as will a loss of the drive train or loss of fuel feed.

(3) The Kill Expression. The relationship between component loss and an aircraft kill can be expressed using the logical AND and OR statements. This logical expression is called the kill expression. As an example, a portion of the kill expression for the kill tree depicted in Figure 2.9 is given by:

[(Pilot .OR. Pilot Controls) .AND. (Copilot .OR. Copilot Controls)] .OR. (Engine 1 .AND. .OR. (Drive Train) .OR. etc.

C. VULNERABILITY ASSESSMENT

1. Defining a Vulnerability Assessment

A vulnerability assessment is the process of determining numerical values for the measures of vulnerability. Target vulnerability analysis is a scientific discipline involving both experimental and analytical processes. Preliminary theories which attempt to describe the response of a target to a particular threat is accomplished during the analysis. Experimentation provides the data used to corroborate or repudiate the theories developed during analysis. Target vulnerability concepts are based on fundamental physical principles. These principles include the theory of: hydraulic ram, ignition, crack propagation, engine response to fuel ingestion, and structural response to blast and penetration. Vulnerability assessments may be carried out entirely "by hand", or one or more computer programs may be used. Assessments are usually conducted to help the designer evaluate the vulnerability of a design, or

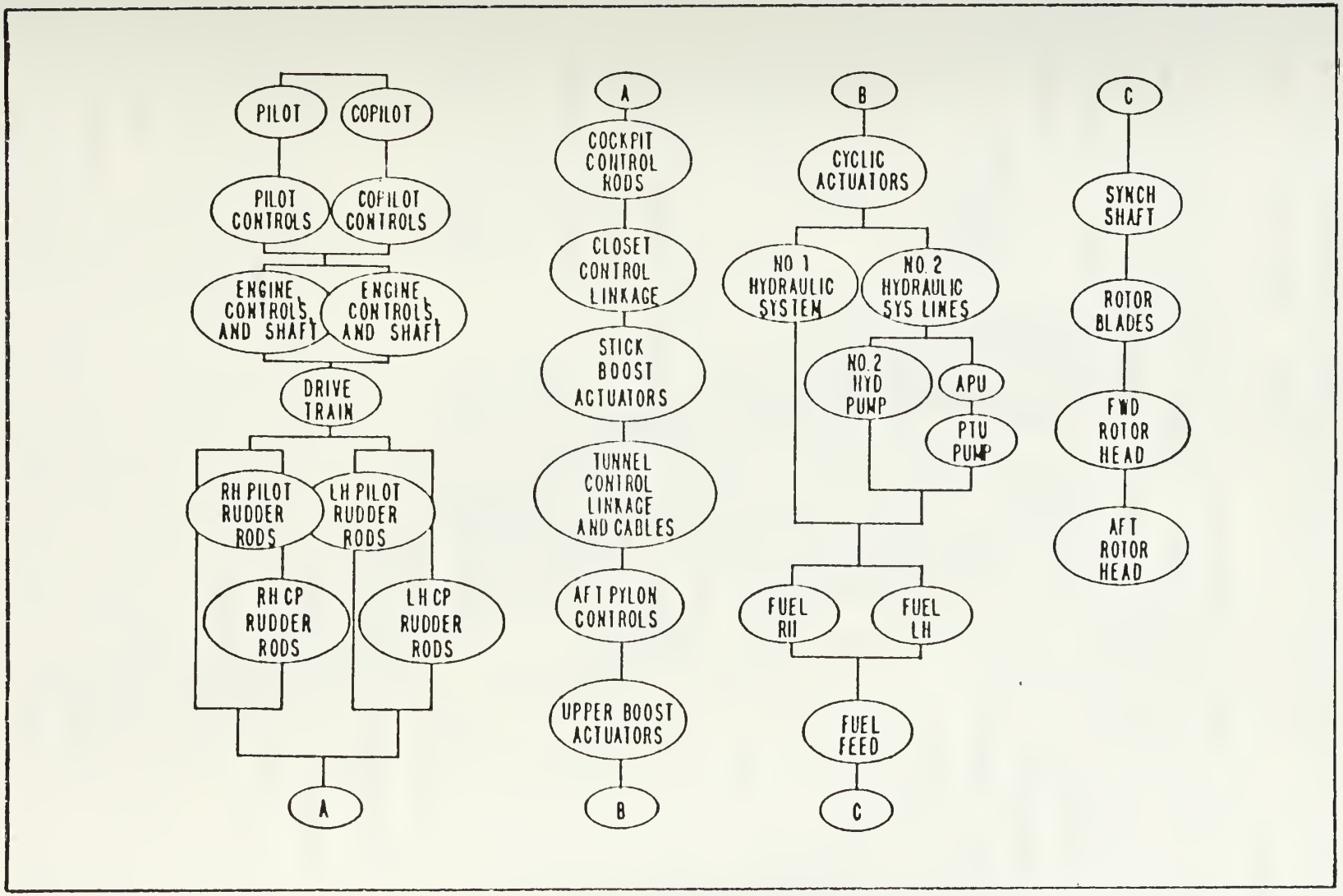


Figure 2.9 Sample Kill Tree for a Two Engine Two Pilot Helicopter

more importantly, by the military to predict the response of targets to a particular threat before threat and target engagement.

A vulnerability assessment is carried out at one of three general levels of detail. These levels are estimates, evaluations, and analyses. Most assessments consider five fragment impact velocities from 1000 to 10,000 ft/sec and use as a minimum, the six cardinal aspects. For a minimum level assessment, the six major aspects shown in Figure 2.10 are usually considered for each kill level. The 26 aspects depicted in Figure 2.11 are usually considered when a more detailed analysis or a computer analysis is performed. Estimates typically use simple equations for the aircraft vulnerability measures that are functions of a few major parameters. These equations are referred to as regression equations if they are fitted to historical data on several aircraft or to the results from engineering studies. Evaluations are more detailed than estimates and may include such items as the individual component locations, sizes, and vulnerability measures. Analyses are very detailed assessment studies that use specific technical and functional information about the components and their vulnerability. Analyses are usually conducted on a digital computer using complex geometric target models.

2. Vulnerability Measures

Because of the diverse nature of the hostile environment in which aircraft operate, the measures of the vulnerability of an aircraft vary with the type of threat encountered. For example, if a hit on the aircraft must occur in order for a threat to be effective, such as a small arms projectile and a contact-fuzed high explosive warhead, one measure of vulnerability is the conditional probability the aircraft is killed given a random hit on the aircraft,

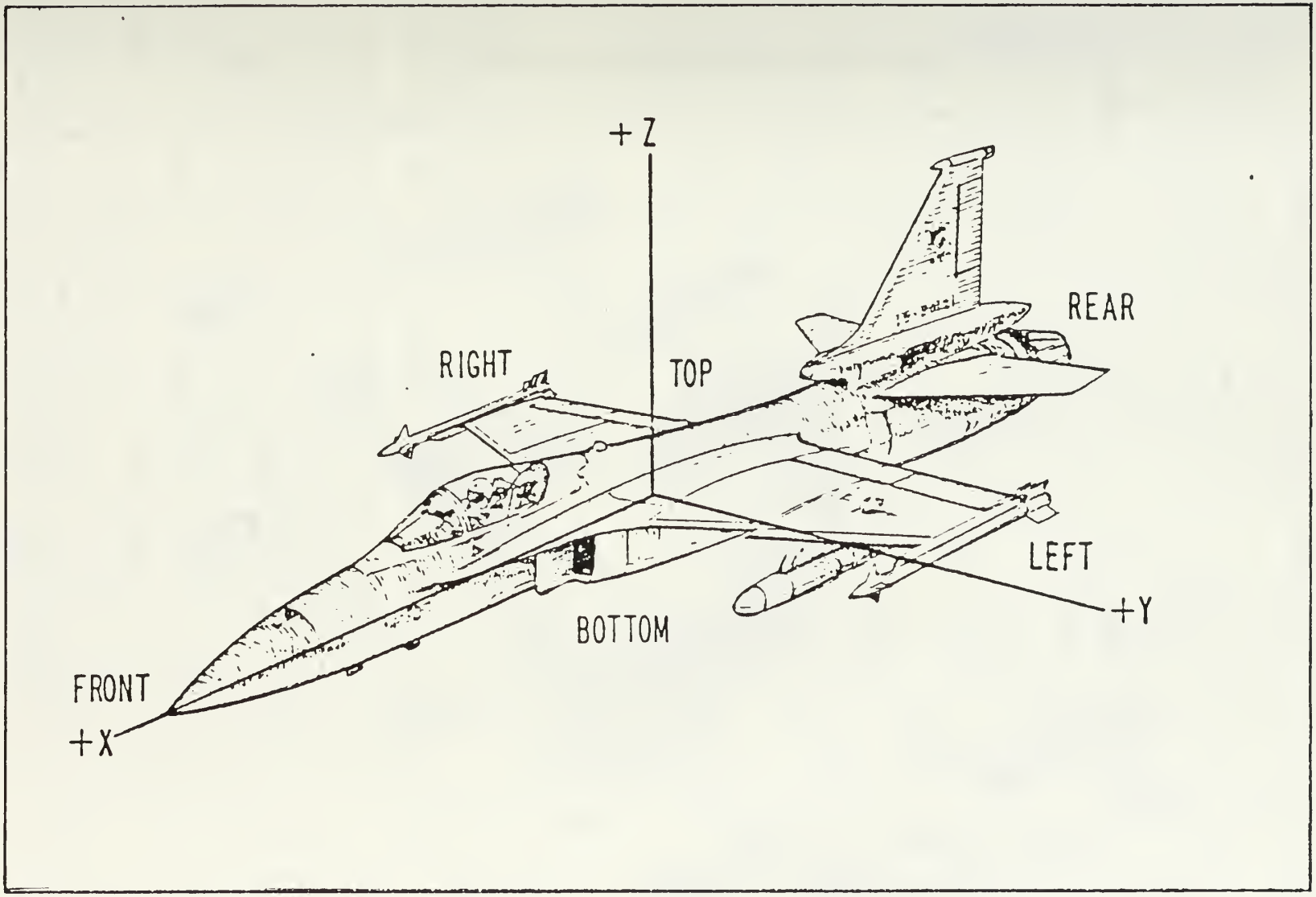


Figure 2.10 Six Aircraft Aspects for a Minimum Assessment

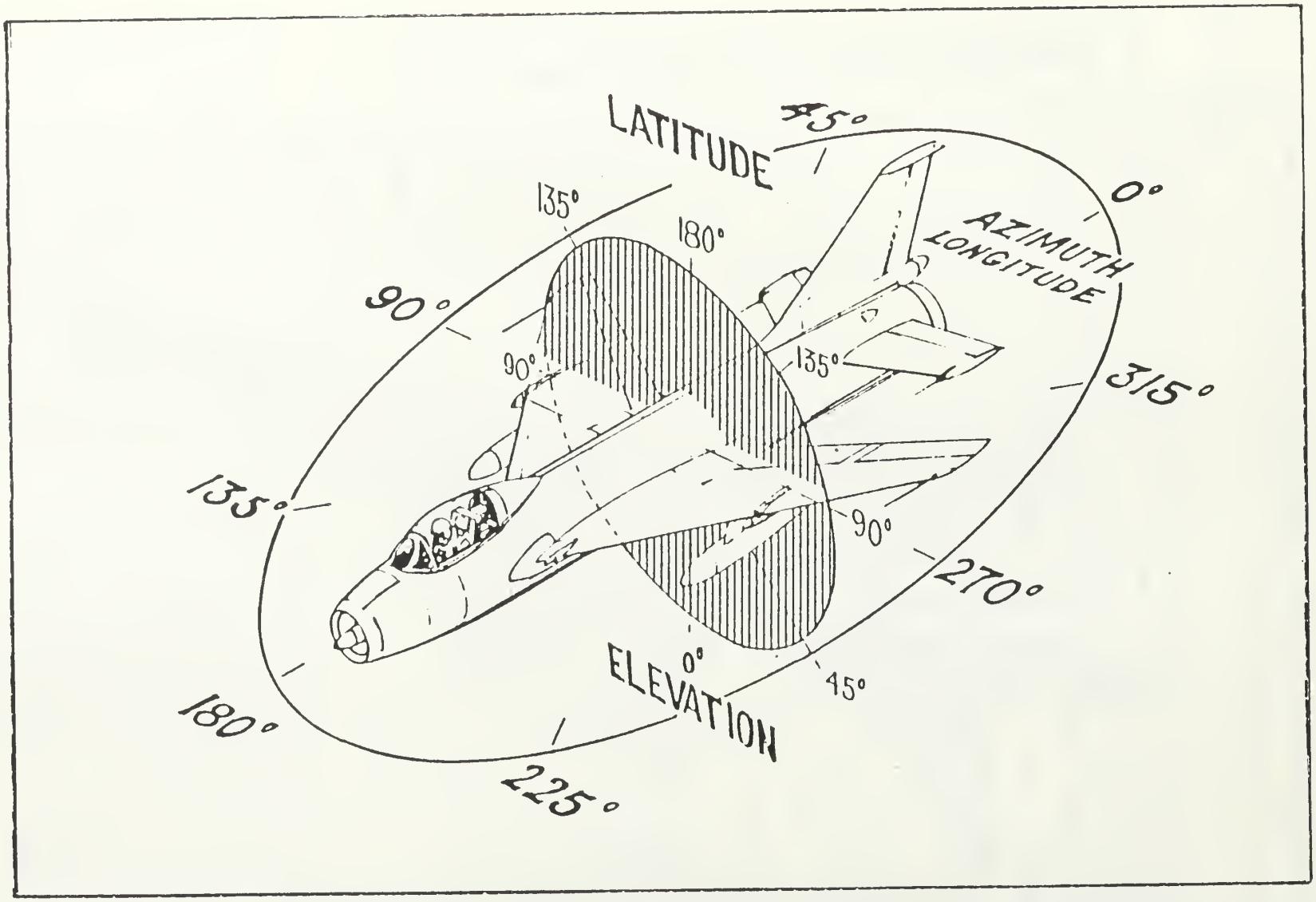


Figure 2.11 26 Aircraft Aspects for a Detailed Analysis

$P_{K/H}$. Another measure of vulnerability to impacting damage mechanisms is the aircraft's vulnerable area, A_V. Vulnerable area is a theoretical, non-unique area presented to the threat which, if hit by a damage mechanism, would result in an aircraft kill. On the other hand, when damage is caused by the effects of a nearby high explosive detonation, the vulnerability may be expressed in the form of a $P_{K/D}$ (probability of kill given a detonation) envelope. This envelope represents a kill probability contour about the aircraft on which a specified detonation will result in a certain probability of aircraft kill. If only the blast from the exploding warhead is considered, the envelope represents the aircraft's vulnerability to external blast. A measurement which is becoming more important relates to aircraft vulnerability to a laser threat. Laser vulnerability can be measured by the probability of kill, given a specific power laser lock-on for a specified period of time, $P_{K/Lo}$.

3. General Requirements

Certain required elements of a vulnerability assessment are common to all studies, regardless of the type of threat considered. These elements are: (1) a selection of the aircraft kill levels or categories to be assessed, (2) an assembly of the technical and functional descriptions of the aircraft, (3) a determination of the critical components of the aircraft, (4) a selection of the specific threats the system will encounter, (5) an analysis to identify the type and amount of damage required to kill each critical component, and (6) the computation of the appropriate vulnerability measures for the components and the aircraft based upon the threat selected. The first three steps of the assessment have been described in the preceding section. A presentation of the last three steps follows.

a. Threat Selection

Because of the many diverse and terminal effects of the various damage mechanisms, each vulnerability assessment is usually made considering either a specific threat or a specific damage mechanism. Mechanisms which may cause damage to an aircraft may be classified as: kinetic energy penetrators such as projectiles and fragments, internal and external blast, pyrophorics, shaped charged jets, focused blast fragments and lasers.

Kinetic energy penetrators include, but are not limited to, ball projectiles, armor piercing projectiles, and fragments. These penetrators cause damage to aircraft components during penetration and perforation. Armor piercing projectiles are constructed with a hardened core which enhances the penetration characteristics of these projectiles over those of the ball type. Most small armor piercing projectiles are prone to tumbling after impact thereby increasing the size of the hole that they tear in the internal components.

Armor piercing incerdiary projectiles contain an incendiary mix encased within the nose of the projectile ahead of a hardened case. Upon impact the jacket peels off, and the incendiary material flashes as the projectile core penetrates the target.

Large high explosive projectiles and missiles can be equipped with influence or command fuzes causing them to detonate nearby an aircraft. These projectiles or missiles have the capability of inflicting damage from external blast effects, fragment impact effects, or a combination of both.

Many smaller high explosive projectiles are equipped with delay fuzes. These fuzes initiate upon contact with the aircraft skin and detonate the projectile

within the aircraft. This high explosive detonation produces internal blast and fragmentation effects. These fragments usually are smaller and slower than the larger high explosive projectiles but the proximity of the detonation to the critical components results in a spray of many fragments impacting the components. This combination of internal blast and fragmentation effects is especially lethal to lightly constructed components such as oil and fuel lines, oil and fuel tanks, hydraulic tanks, and the aircrew.

b. Critical Component Kill Criteria

Once the set of critical components for a given aircraft has been identified, the damage or kill criteria for each of the failure modes of these components must be determined for the selected threats. Damage criteria for a critical component is the level of damage required for a preestablished degradation of the performance of the component. Thus, a kill criterion is the specific descriptive characteristics or quantification of a component failure. Some examples of critical component kill criteria are: the amount of material that must be removed from a drive shaft for failure, requirements for failure of a structural member, the amount of damage required to incapacitate a system of gears, the minimum diameter of hole in a fuel tank or line for engine starvation within a specified time period, etc. Very few kill criteria are precisely known, nor can they easily be determined. Battle damage reports are an important source of component damage effects information. The results of tests conducted on all types of aircraft components and subsystems provide another increasingly important and expanding source of data. Data is required for each critical component that allows for the effects of encounter parameter variations, such as fragment

or penetrator striking velocity, obliquity angle, shape and mass of the fragment or penetrator, to be estimated for each kill category and level being assessed. In the case of externally detonating warheads, high explosive projectiles, and contact-fuzed missiles, miss distance boundaries relating $P_{K/H}$ to burst points are established to assess blast effects of these threats.

The major result of this task is the specification of numerical values for the kill criteria for each failure mode for each critical component for each threat to be considered. Three specific kill criteria are currently in use for the impacting damage mechanisms. They are the probability of component kill given a hit, the area removal criterion, and the energy density criterion. There is a fourth criterion which applies to the blast damage mechanisms.

(1) The Probability of Kill Given a Hit Function. The $P_{k/h}$ function defines the probability of a component kill when impacted by a fragment or penetrator. This criterion can be presented graphically as a function of the mass and velocity of the damage mechanism, or it can be expressed in an analytical form. Figure 2.12 is a sample of $P_{k/h}$ data for a flight control rod.

The $P_{k/h}$ criterion is normally used for components that can be killed by a single hit, such as crew members, control rods, electronic equipment, and servoactuators. These components are sometimes referred to as single fragment vulnerable components. It can also be used for some of the larger components, such as engines and fuel tanks. In this case, the volume of the large component is usually divided into several smaller volumes, and a different numerical value of $P_{k/h}$ is assigned to each volume. For example, a fuel tank could be divided into the ullage, fuel, and external void spaces, and a turbojet

engine could be subdivided into the major sections illustrated in Figure 2.13.

The determination of the $P_{k/h}$ for each component or part of a component is a very difficult undertaking. It requires a combination of critical component analysis data and sound engineering judgment. Although

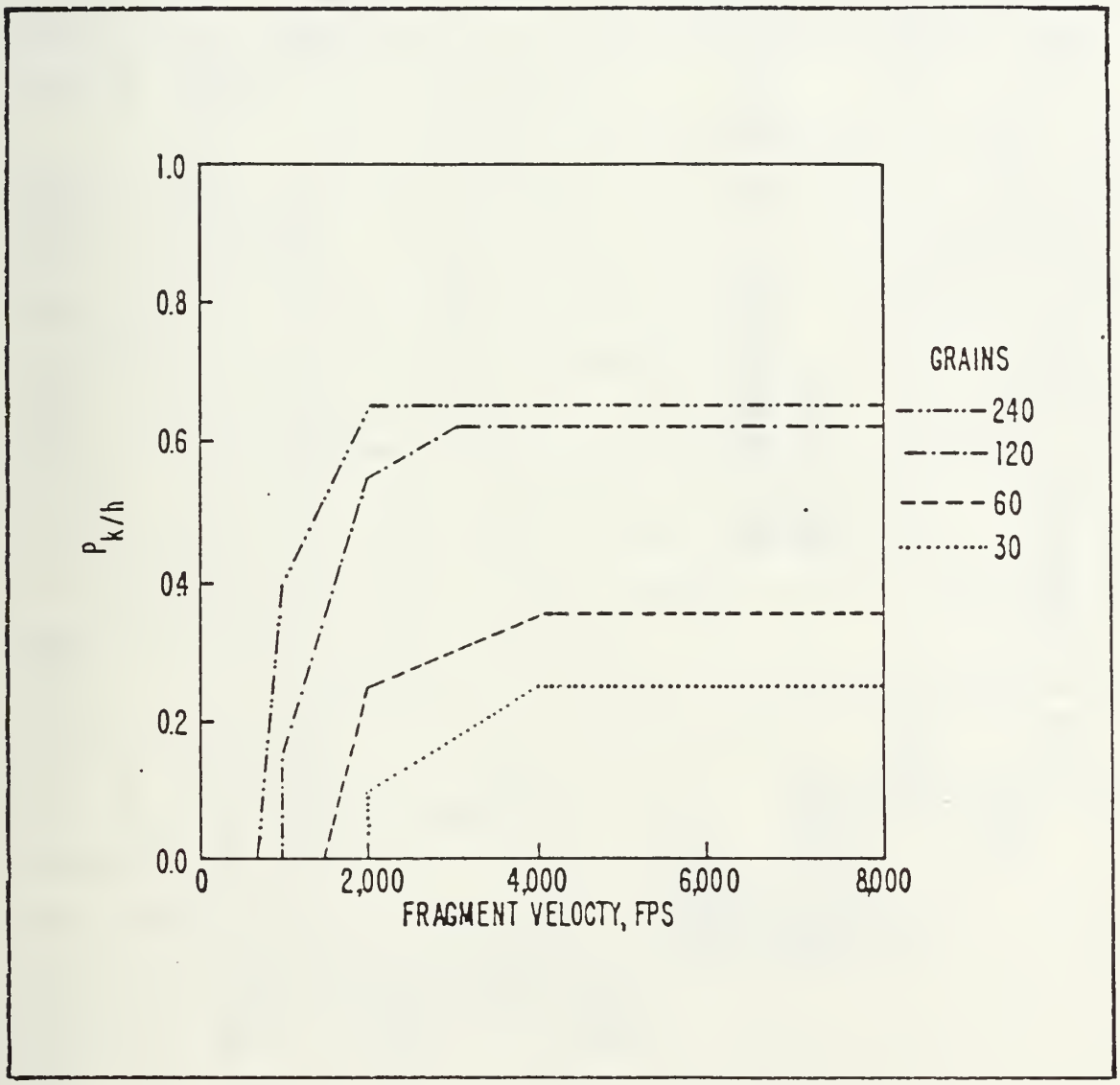


Figure 2.12 Typical $P_{k/h}$ Data for a Flight Control Rod

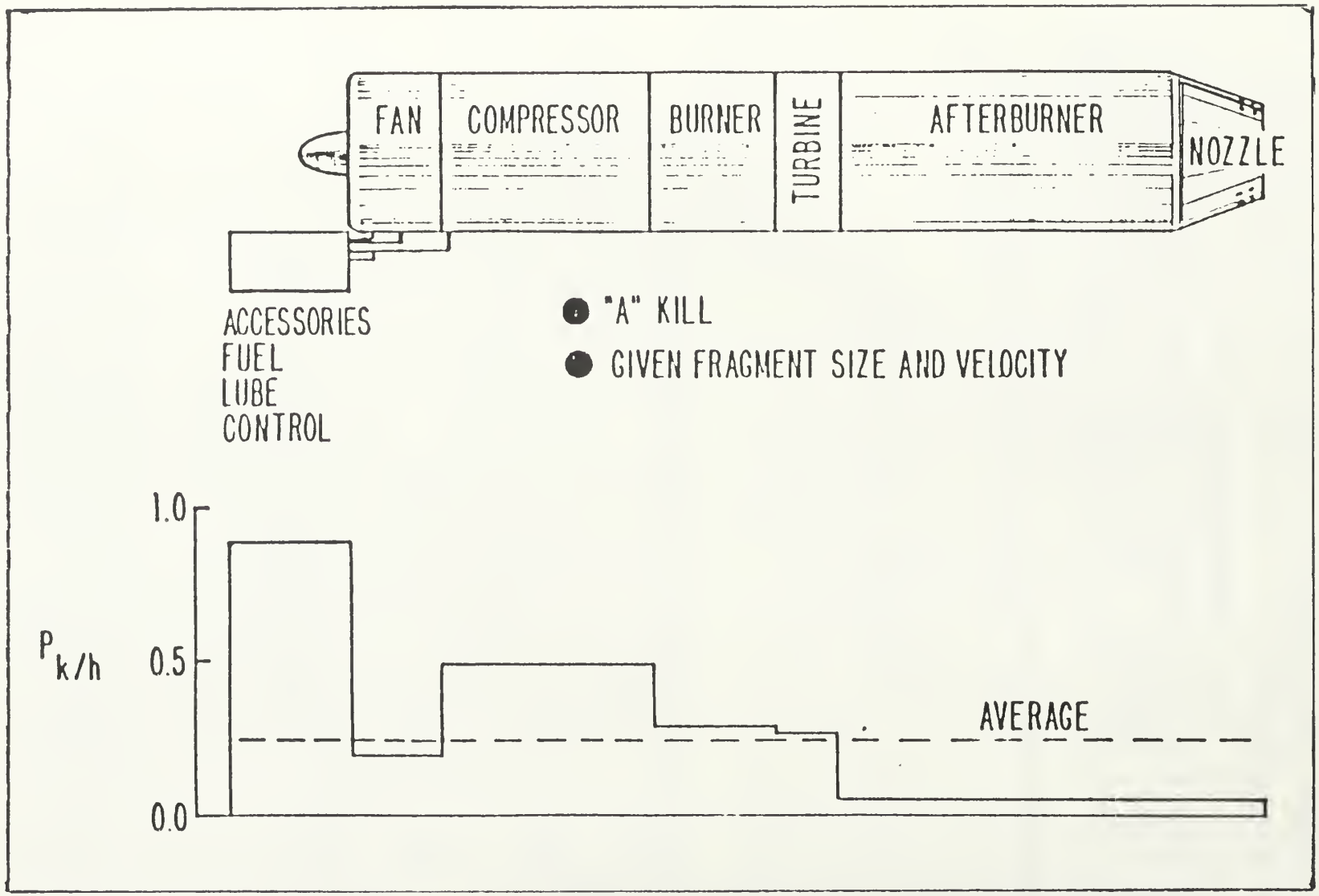


Figure 2.13 Engine $P_{k/h}$ Data

limited gunfire testing provides some insight intc the effects cf projectile and fragment damage potential, there is no universal methodology fcr arriving at a numerical value fcr $P_{k/h}$. The larger ccmponents, such as the fuel tanks and engines, are especially difficult to evaluate due to the multitude of local environments, the constantly changing operation conditions, and the many different failure modes. Numbers for $P_{k/h}$ are eventually assigned based upon a combination of empirical information, engineering judgment, and experience.

The location of the component inside the aircraft will have an influence on its ultimate numerical probability of kill given a hit, but not on its $P_{k/h}$ function. Components located behind thick structures cr dense equipment packs will receive a level of protection due to the slowdcwn of the damage mechanism as it attempts tc penetrate the shielding ccmponents. The numerical value of the $P_{k/h}$ for the lowered velocity of impact will generally be less than the $P_{k/h}$ fcr the impact of a penetrator cr fragment that was not slowed down. Other considerations, such as spall and fragment breakup caused by the intervening components also becomes important.

The area removal criterion defines a specific amount of area that must be removed from a ccmponent in order to kill that ccmponent. This criterion is applicable to large penetrators, such as rods, and to the closely spaced hits from many fragments. The total ccmponent damage from a ccllection of closely spaced hits can be greater than the sum cf the individual damages from the same number of widely spaced hits. Cften there is a synergism of damage due to cracking and petalling between the individual holes, and large areas of component structure can be removed or destrcyed. This criterion is used mainly for structural components.

(2) Energy Density. In this criterion, a component kill is expressed in terms of a required minimum component surface area that must be exposed to a minimum threshold level of the kinetic energy density of the impacting damage mechanisms. This criterion is applicable to closely spaced multiple fragment hits and is used for the structural components, as well as other large components, such as the fuel tanks and engines. For some components, there may be a minimum mass of the damage mechanism below which the criterion is not applied.

(3) Blast. The damage criterion for blast is generally the critical values of pressure and impulse on an aircraft surface necessary to cause the specific component damage level associated with the assumed kill level. For example, a dynamic overpressure of two pounds per square inch over the upper surface of a horizontal tail for one millisecond may be sufficient to cause crushing of the skin, leading to a loss of stiffness and the inability to support the flight loads. Although this criterion is usually applied to the structural components and control surfaces, the effects of the blast can extend into the interior of the aircraft and can damage electrical wiring, hydraulic lines, fuel tank walls, and other internal components located close to the aircraft skin.

c. Computation of the Vulnerability Measures

The procedures used to compute the vulnerability of an aircraft and its components to an externally detonating high explosive warhead and non-explosive penetrators or fragments, to an internally detonating high explosive warhead, and to lasers are described in the following three sections.

D. VULNERABILITY TO EXTERNALLY DETONATING WARHEADS

Vulnerability of an aircraft to an externally detonating high explosive warhead is usually analyzed in two steps. The first step is a determination of the aircraft's vulnerability to the blast, and the second step analyzes the aircraft's vulnerability to the fragments and penetrators. In addition, both analyses must consider the encounter scenario between the aircraft and the missile at the time of warhead detonation. For this reason, this section has been divided into the following two subsections: the effect of fragments and penetrators striking an aircraft, and blast. Shortly after detonation, the blast front precedes the fragments. Eventually, the fragments pass through the blast front because the fragment velocity decay is less than the blast front velocity decay. The overpressure caused by the warhead detonation can cause serious damage to aircraft structure and components. Using the conditions of the encounter scenario, the blast is analyzed for impulse and overpressure to determine iso-damage contours for an aircraft kill. If a detonation occurs close enough to inflict serious blast damage, the fragments most likely will cause much more damage than that caused by the blast. In the fragments and penetrators subsection, the vulnerability to fragments and penetrators is computed for both the single hit case and the multiple hit case. A typical encounter is shown in Figure 2.14.

1. Effect of Fragments/Penetrators Striking an Aircraft

The vulnerability of an aircraft to a single impacting penetrator or fragment is usually expressed as a total vulnerable area, A_V, or as a probability of aircraft kill given a random hit on the aircraft, $P_{K/H}$. The vulnerable area concept is applicable to both the aircraft and to

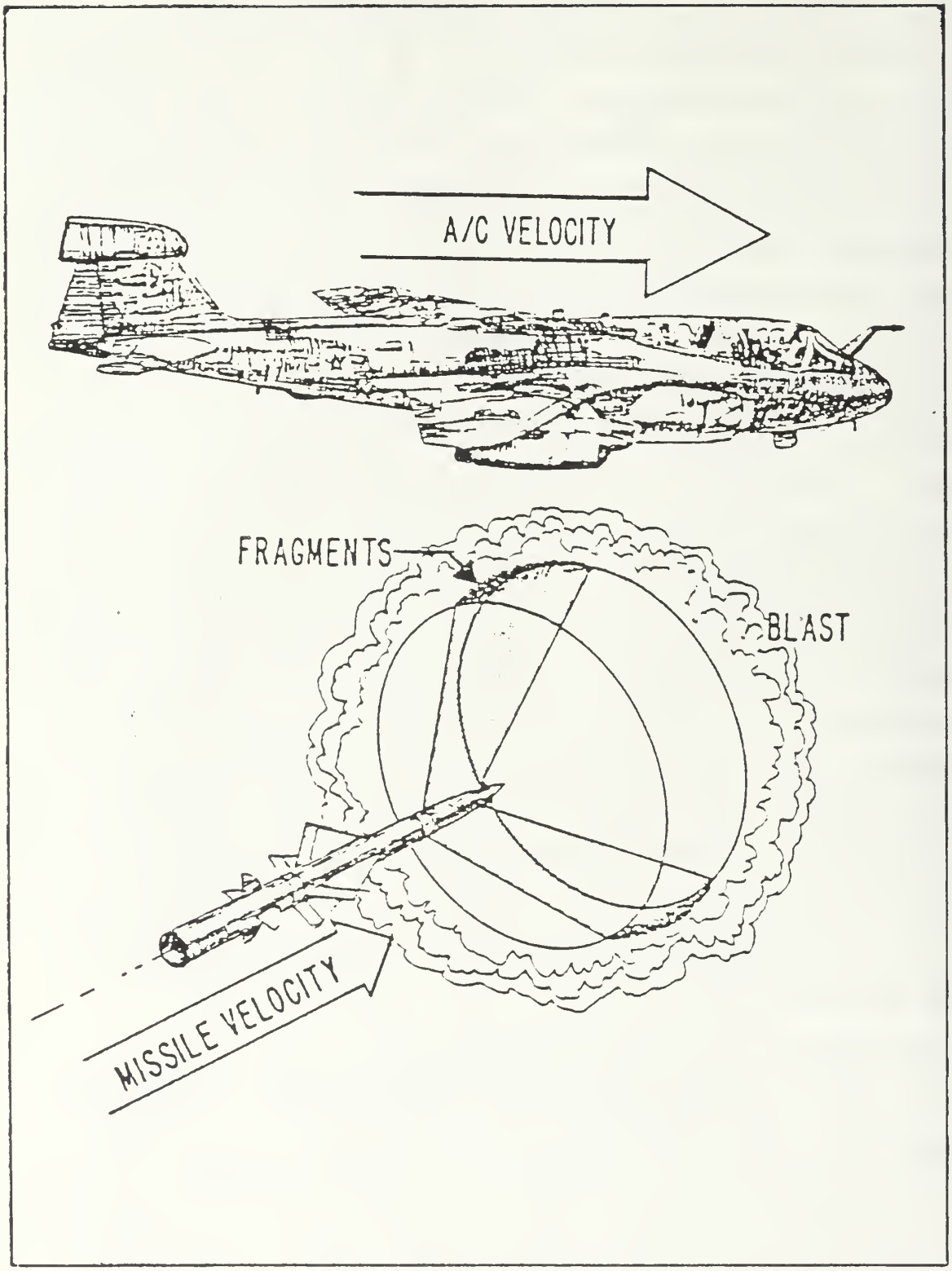


Figure 2.14 Typical Aircraft Encounter With An Externally Detonating HE Warhead

its critical components. The vulnerable area of the typical ith component is denoted by A_{v_i}, and the component kill criterion used is the probability of kill given a hit, P_{k/h_i}. To assist the reader in keeping track of the notation used in this presentation, the variable and subscript definitions are summarized in Table 2.

TABLE 2

Vulnerability Assessment Variable Definitions

Definition	Variable
Probability of killing the ith component given a hit on the ith component	P_{k/h_i}
Probability of killing the ith component given a hit on the aircraft	P_{k/H_i}
Probability of killing the aircraft given a hit on the aircraft	$P_{K/H}$
Vulnerable area of the ith component	A_{v_i}
Vulnerable area of the aircraft	A_V
Presented area of the ith component	A_{p_i}
Presented area of the aircraft	A_P

Note that a distinction is made between component and aircraft designated variables by using lower and upper case subscripts, respectively.

The vulnerable area of the ith component is defined as the product of the presented area of the component in the plane normal to the approach direction of the damage mechanism (the shotline), A_{p_i}, and the probability of kill of the component given a hit on the component, P_{k/h_i}. Thus,

$$A_{v_i} = A_{p_i} \times P_{k/h_i} \qquad (2.1)$$

Since both A_{p_i} and P_{k/h_i} are generally functions of the threat direction or aspect, the vulnerable area will also vary with aspect. In the discussion that follows, it is important to recall that:

$$P_{S/H} = 1 - P_{K/H} \tag{2.2}$$

where $P_{S/H}$ is the probability the aircraft or component survives the hit, and $P_{K/H}$ is the probability of killing that aircraft or component.

The kill probability of the ith component given a random hit on the aircraft, P_{k/H_i}, is:

$$P_{k/H_i} = P_{h/H_i} \times P_{k/h_i} \tag{2.3}$$

where P_{h/H_i} is the probability the component is hit given a hit on the aircraft, and P_{k/h_i} is defined as the probability the component is killed given a hit on the component. From Equation 2.2, it follows that:

$$P_{s/H_i} = 1 - P_{k/H_i} \tag{2.4}$$

Using Equations 2.1 and 2.3, and solving for P_{h/H_i} gives:

$$P_{h/H_i} = A_{p_i} / A_P \tag{2.5}$$

where A_P is the presented area of the total aircraft in the plane normal to the threat direction. Substituting Equations 2.5 and 2.1 into Equation 2.3 determines, for any random hit on the aircraft, the probability the ith component is killed, and is given by:

$$P_{k/H_i} = A_{v_i} / A_P \tag{2.6}$$

The numerical value for P_{k/H_i} depends upon the presented area of the critical component, A_{p_i}, and of the aircraft, A_p, and upon the component kill criterion, P_{k/h_i}. The presented area of the critical components and of the aircraft can be obtained from the available technical description of the aircraft. The procedure for determining the numerical value for P_{k/h_i} is described in the presentation on the critical component kill criteria given above.

In this assessment, a component is assumed to be either operating and performing all of its functions or killed. No degradation of component capabilities is considered due to a hit, and no compounding of component damage is recognized. Although these assumptions are usually made in a vulnerability assessment, they are not necessary. Theoretically, only the component hit can be killed. Although the kill of adjacent components, perhaps by fire or explosion, is not directly considered here, a procedure for indirectly accounting for kills of adjacent components will be described later.

Now that the concepts of vulnerable area and the probability of kill given a hit have been explained, the scenario must be considered. In any given combat engagement, the aircraft will either not be hit, it will be hit only once, or the aircraft will be hit more than once. The no hit situation is not of interest here. The location on the presented area of the aircraft of the single hit and of multiple hits is assumed in the vulnerability assessment to be a random distribution, with each damage mechanism having the same approach or attack direction. In other words, the assumption is usually made that the enemy has no capability to direct hits to any one particular component, subsystem, or part of the aircraft, and that the damage mechanisms travel along parallel shotlines. The single hit case lays the ground work for the multiple hit case. In both cases,

the influence of nonredundancy and redundancy of components on the vulnerable area must be examined. Overlap of critical components is also an important consideration.

a. Single Hit Vulnerability

Both the nonredundant aircraft model and the redundant aircraft model considered in this section are assumed to receive only one hit. The nonredundant aircraft model is composed of only one of each of the critical components. Thus, the loss of any one critical component will cause the loss of the aircraft. In the redundant aircraft model, some of the critical component functions are duplicated by the same or different components. The effects of overlapping of both nonredundant and redundant critical components are examined. For example, the fact that an engine overlaps (shields) a hydraulic pump will probably decrease the probability of kill of that pump. It is necessary to specify how this overlap effect is quantified for both the nonredundant and redundant aircraft models.

(1) Aircraft Model Composed of Nonredundant Components with No Overlap. This aircraft consists of N critical components whose functions are not duplicated by any other component. The components are arranged in such a way that no components overlap when viewed from a given aspect. Any hit on the aircraft takes place along a shotline that passes completely through the aircraft. Thus, no more than one component can be hit on any one shotline. As an example, Figure 2.15 shows an aircraft consisting of three critical components; a pilot, one fuel tank, and one engine. None of the critical components overlap in the aspect presented in Figure 2.15.

The probability of killing this aircraft, given a random hit on the presented area in Figure 2.15, can be derived using the kill expression and Equations 2.1 and

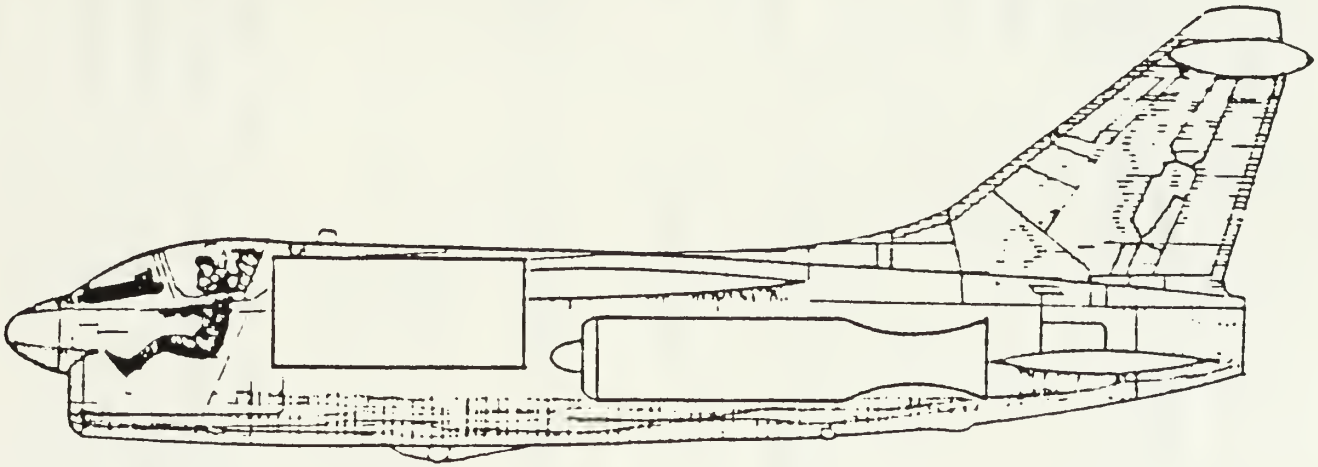


Figure 2.15 Nonredundant Aircraft Model With No Overlap

2.6. For an aircraft composed of N nonredundant critical components, the kill expression is:

$$\text{Kill} = (\text{Nrc1}) \ .\text{OR}. \ (\text{Nrc2}) \ .\text{OR}. \ \ldots \ (\text{NrcN}) \qquad (2.7)$$

where Nrci refers to a kill of the ith nonredundant component. In other words the aircraft kill is defined by the kill of nonredundant component number 1, or nonredundant component number 2, or ..., or nonredundant component number N. Because a kill of any one of the critical components will kill the aircraft, the aircraft will survive only if all of the nonredundant critical components survive. Thus,

$$P_{S/H} = P_{s/H_1} \times P_{s/H_2} \times \ldots \times P_{s/H_N} \qquad (2.8)$$

Using Equation 2.4, Equation 2.8 may be written as:

$$P_{S/H} = (1 - P_{k/H_1}) \times (1 - P_{k/H_2}) \times \ldots \times (1 - P_{k/H_N}) \qquad (2.9)$$

For our model aircraft N=3, and Equation 2.9 becomes:

$$P_{S/H} = 1 - (P_{k/H_1} + P_{k/H_2} + P_{k/H_3}) + (P_{k/H_1} \times P_{k/H_2}) + (P_{k/H_1} \times P_{k/H_3}) + (P_{k/H_2} \times P_{k/H_3}) - (P_{k/H_1} \times P_{k/H_2} \times P_{k/H_3}) \qquad (2.10)$$

Because of the assumption that only the component hit can be killed, and because none of the critical components overlap, the kills of the components are mutually exclusive. This means only one component can be killed by one hit, and the products of the P_{k/H_i} given in Equation 2.10 are not applicable. Therefore, Equation 2.10 simplifies to:

$$P_{S/H} = 1 - (P_{k/H_1} + P_{k/H_2} + P_{k/H_3}) \qquad (2.11)$$

and the probability of killing the aircraft given a hit on the aircraft is just the sum of the individual probabilities of killing each of the critical components given a random hit on the aircraft. This may be written as:

$$P_{K/H} = P_{k/H_1} + P_{k/H_2} + \ldots + P_{k/H_N} \qquad (2.12)$$

Substituting Equation 2.6 into Equation 2.8, and applying the concept of $P_{k/h}$ expressed in Equation 2.1, leads to:

$$P_{K/H} = A_V / A_P \qquad (2.13)$$

where A_V is the summation of vulnerable area of all of the critical components.

For our example aircraft, the kill expression is given by:

$$\text{Kill} = (\text{Pilot}) \text{ .OR. } (\text{Fuel Tank}) \text{ .OR. } (\text{Engine}) \qquad (2.14)$$

From Equation 2.12:

$$P_{K/H} = P_{k/H_p} + P_{k/H_f} + P_{k/H_e} \qquad (2.15)$$

and

$$A_V = A_{v_p} + A_{v_f} + A_{v_e} \qquad (2.16)$$

where the subscripts p, f, and e denote the pilot, the fuel tank, and the engine. From Equation 2.1, the individual component areas are given by:

$$A_{v_p} = A_{p_p} \times P_{k/h_p} \qquad (2.17)$$

$$A_{v_f} = A_{p_f} \times P_{k/h_f} \qquad (2.18)$$

$$A_{v_e} = A_{p_e} \times P_{k/h_e} \qquad (2.19)$$

For illustration, a numerical example is presented in Table 3.

TABLE 3

Nonredundant Model Without Overlap
With Mutually Exclusive Kill Modes

Critical Component	A_{p_i}	x	P_{k/h_i}	=	A_{v_i}	P_{k/H_i}
Pilot	4 ft²		1.0		4 ft²	.0133
Fuel	60 ft²		0.3		18 ft²	.0600
Engine	50 ft²		0.6		30 ft²	.1000

A_P = 300 ft² A_V = 52 ft² $P_{K/H}$ = .1733

The kill of one critical component due to damage caused by a hit on another critical component and the consideration of multiple kill modes of a critical component can be indirectly accounted for, in this model, by increasing the numerical value of the kill criterion for the component hit. Consider two failure modes that are not mutually exclusive, that is, both can occur with a single hit. For example, suppose the probability the fuel tank of an aircraft is destroyed by a fire when the fuel tank is hit, is taken as 0.3. Suppose further that the probability that the fuel tank is penetrated and that hydraulic ram damage causes fuel to be dumped into the air inlet and ingested by the engine, leading to an aircraft loss, is taken as 0.1. The aircraft will survive a hit in the fuel

tank only if there is neither a fire nor any fuel ingestion. The probability that neither of these failure modes occur when the fuel tank is hit is given by the product of the probability that there is no fire (1 - 0.3), and the probability that there is no fuel ingestion kill of the engine (1 - 0.1), which is 0.63. Therefore, the probability that there will be a fire kill and/or a fuel ingestion kill, given a hit on the fuel tank, is given by (1 - 0.63), or 0.37. A numerical example is presented in Table 4.

TABLE 4

Nonredundant Model Without Overlap With Mutually Inclusive Kill Modes

Critical Component	A_{p_i}	x	P_{k/h_i}	=	A_{v_i}	P_{k/H_i}
Pilot	4 ft²		1.0		4.0 ft²	.0133
Fuel	60 ft²		0.37		22.2 ft²	.0740
Engine	50 ft²		0.6		30.0 ft²	.1000

A_P = 300 ft² A_V = 56.2 ft² $P_{K/H}$ = .1873

Note that in this case the $P_{k/h}$ is not the sum of the two individual kill probabilities because there can be both a fire kill and a fuel ingestion kill on the one hit. Comparing Table 3 with Table 4 shows that by accounting for the additional failure mode of fuel ingestion by the engine increases the fuel tank $P_{k/h}$ with the accompanying change in component and aircraft vulnerable area, and the component and aircraft probability of kill. This same procedure can be used to compute the P_{k/h_i} due to multiple failure modes of one critical component.

(2) Aircraft Model Composed of Nonredundant Components with Overlap. The aircraft model will now be expanded by allowing two or more critical components to overlap in an arbitrary manner. An example aircraft is presented in Figure 2.16. There can be any number of critical components along a shotline within the overlap area. For the aircraft to survive a hit along a shotline within a region of N overlapping critical components, each critical component along that shotline must survive. The probability the aircraft survives a hit on the overlap region, P_{s/h_o}, is given by:

$$P_{s/h_o} = P_{s/h_1} \times P_{s/h_2} \times \ldots \times P_{s/h_N} \tag{2.20}$$

Because two or more critical components in the overlap region can be killed by one hit, the kills of more than one component are not mutually exclusive. In this case, Equation 2.11 is not valid, and Equation 2.20 must be used for hits in the overlap region. For the aircraft illustrated in Figure 2.16, the probability the aircraft survives a hit on the overlap region is given by Equation 2.21, where the subscripts f and e refer to the fuel tank and the engine.

$$P_{s/h_o} = P_{s/h_f} \times P_{s/h_e} \tag{2.21}$$

If the overlap area, A_{p_o}, is now considered as a separate component, the probability of kill given a hit on the component may be written as:

$$P_{k/h_o} = 1 - P_{s/h_o} \tag{2.22}$$

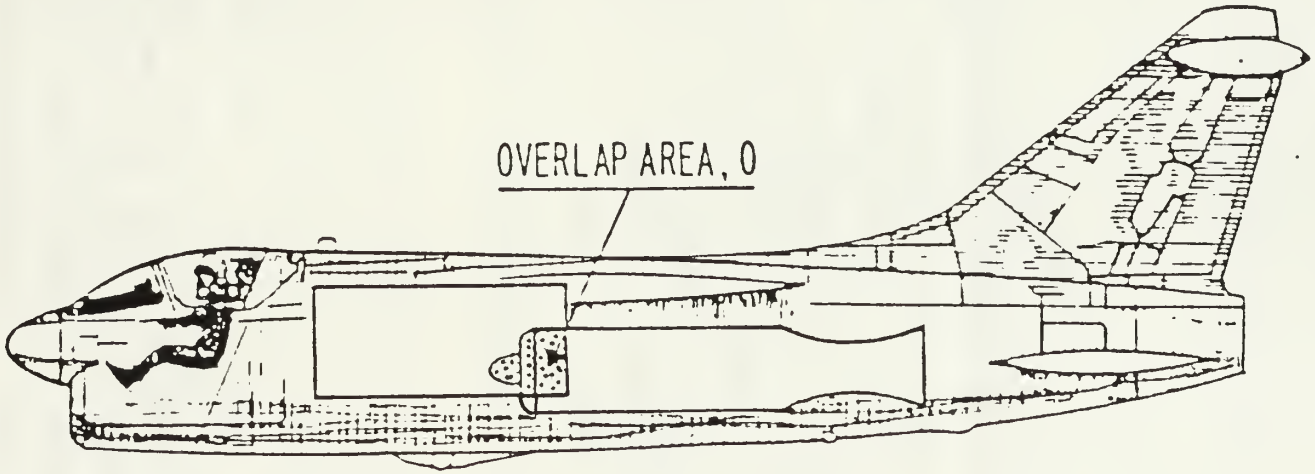


Figure 2.16 Nonredundant Aircraft Model With Overlap

and the vulnerable area of the overlap area, A_{v_o}, is given by:

$$A_{v_o} = A_{p_o} \times P_{k/h_o} \tag{2.23}$$

Substituting Equation 2.21 into Equation 2.23, and using Equation 2.4, gives:

$$P_{k/h_o} = 1 - [(1 - P_{k/h_f}) \times (1 - P_{k/h_e})] \tag{2.24}$$

It is assumed for this example that the overlap area in Figure 2.16 is 10 ft², the fuel tank $P_{k/h}$ is 0.3, the engine $P_{k/h}$ is 0.6, and all other areas are the same as used in the nonoverlapping example. The fuel is assumed to slow the damage mechanism down, but not enough to change the engine $P_{k/h}$. Because the $P_{k/h}$ values are the same as in the nonoverlapping example, any reduction in the vulnerable area of the aircraft is due only to the component overlap. Thus, in the overlap region:

$$P_{k/h_o} = 1 - [(1-0.3) \times (1-0.6)] = 0.72 \tag{2.25}$$

and

$$A_{v_o} = 10 \times 0.72 = 7.2 \text{ ft}^2 \tag{2.26}$$

according to Equations 2.23 and 2.24.

The vulnerable area of the overlap area contributes to the aircraft vulrerable area. However, overlapping also requires that the overlap area be subtracted from the total presented area of each overlapping component contributing to the overlap. The component area outside of the overlap is treated in the usual way. Table 5 illustrates computing the vulnerable area of an aircraft with

overlapping components. Note that locating two of the critical components such that one overlaps the other reduces the aircraft vulnerable area form 52 ft² to 50.2 ft². This is an example of how location of the critical components can reduce the aircraft's vulnerable area.

TABLE 5

Vulnerable Area Computation for Nonredundant Model With Overlap and No Engine Fire

Critical Component	A_{p_i}	x	P_{k/h_i}	=	A_{v_i}
Pilot	4 ft²		1.0		4.0 ft²
Fuel	60-10=50 ft²		0.3		15.0 ft²
Engine	50-10=40 ft²		0.6		24.0 ft²
Overlap Area	10 ft²		0.72		7.2 ft²
				A_V =	50.2 ft²

The net effect of component overlap can be a desirable reduction in aircraft vulnerable area provided the damage inflicted by the hit in the overlap area does not cause other problems. For example, consider a shotline through the fuel tank that overlaps the engine. Fuel could leak from the punctured tank onto hot engine parts, causing a fire. In this instance, the probability the engine is killed by the hit would probably be higher than 0.6. An example of the computation of aircraft vulnerable area, assuming the possibility of an engine fire, is given in Table 6. The overlapping area is assumed to be 10 ft², the fuel tank $P_{k/h}$ is assumed to be 0.3, and the $P_{k/h}$ for the

engine is taken as 0.9 because an engine fire is assumed to occur nearly always due to a hit on the overlapping fuel tank. Then,

$$P_{k/h_o} = 1 - [(1 - 0.3) \times (1 - 0.9)] \qquad (2.27)$$

and the aircraft's vulnerable area increases to 52.3 ft².

TABLE 6

Vulnerable Area Computation for Nonredundant Model With Overlap and an Engine Fire

Critical Component	A_{p_i}	x	P_{k/h_i}	=	A_{v_i}
Pilot	4 ft²		1.0		4.0 ft²
Fuel	60-10=50 ft²		0.3		15.0 ft²
Engine	50-10=40 ft²		0.6		24.0 ft²
Overlap Area	10 ft²		0.93		9.3 ft²
				A_V =	52.3 ft²

Comparing the aircraft's vulnerable areas given in Tables 3, 5, and 6 reveals that overlapping the engine with the fuel tank reduces the vulnerable area from 52 ft² to 50.2 ft², provided no fire can occur. If a fire is likely to occur, the vulnerable area increases from 52 ft² to 52.3 ft². Thus, overlapping nonredundant critical components can reduce vulnerability provided that no undesirable secondary kill modes occur.

Another facet of the overlap situation is the change in the vulnerable area of the overlap area that

occurs when one of the components along a shotline has its vulnerability reduced by use of a vulnerability reduction technique. For example suppose the $P_{k/h}$ of the overlapping fuel tank is reduced from 0.3 to 0.0. The vulnerable area of the overlap section, with the possibility of a fire, is reduced from 7.2 ft² to 6.0 ft². This reduction appears to conflict with the fact that 10 ft² with a $P_{k/h}$ of 0.3, and a vulnerable area of 3.0 ft², has been made invulnerable. The reason for this apparent contradiction is the fact that the fuel tank is only one of two overlapping components. Generally, when the vulnerability of one component is reduced, the vulnerability of another component along the shotline will become more important. The vulnerable area of each component along the shotline is referred to as the true vulnerable area, and the components contribution to the overlap vulnerable area is referred to as the incremental vulnerable area. Using the data of Table 5, the true vulnerable areas are 3 ft² and 6 ft² for the overlapping fuel tank and engine areas, and the incremental vulnerable areas of the two overlapping components are 1.2 ft² and 4.2 ft² respectively.

(3) Aircraft Model Composed of Redundant Components with No Overlap. The nonredundant aircraft model described above will now be expanded by adding a second, separated engine, as shown in Figure 2.17. The second engine is assumed to have the same presented area as the first engine, 50 ft², but its $P_{k/h}$ is taken as 0.7 because of the presence of an additional accessory drive. For the purpose of comparison, the aircraft's presented area will remain 300 ft². The kill expression for this model aircraft is:

Kill = (Pilot) .OR. (Fuel Tank) .OR. [(Engine 1) .AND. (Engine 2)] (2.28)

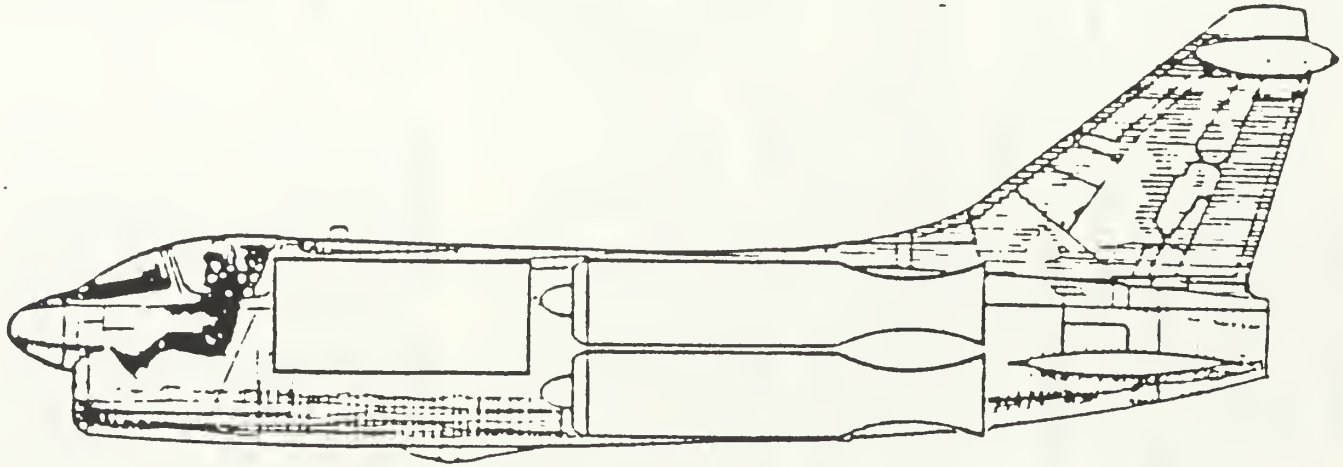


Figure 2.17 Redundant Aircraft Model With No Overlap

Table 7 presents the values for the vulnerability parameters. The Equation for the probability of aircraft survival given a random hit on the aircraft is:

$$P_{S/H} = P_{s/H_p} \times P_{s/H_f} \times [1 - (P_{k/h_{e1}} \times P_{k/h_{e2}})] \quad (2.29)$$

which can be rewritten as:

$$P_{S/H} = (1 - P_{k/H_p}) \times (1 - P_{k/H_f}) \times [1 - (P_{k/H_{e1}} \times P_{k/H_{e2}})] \quad (2.30)$$

TABLE 7

Redundant Aircraft Model Without Overlap

Critical Component	A_{p_i}	x	P_{k/h_i}	=	A_{v_i}	P_{k/H_i}
Pilot	4 ft²		1.0		4 ft²	.0133
Fuel	60 ft²		0.3		18 ft²	.0600
Engine 1	50 ft²		0.6		30 ft²	.1000
Engine 2	50 ft²		0.7		35 ft²	.1167

$A_p = 300$ ft² $A_V = 22$ ft² $P_{K/H} = .0733$

Equation 2.30 says that the aircraft is killed if the pilot is killed, or if the fuel tank is killed, or if both engines are killed. Carrying out the multiplication indicated in Equation 2.30 leads to:

$$P_{S/H} = 1 - (P_{k/H_p} + P_{k/H_f}) + (P_{k/H_p} \times P_{k/H_f}) - (P_{k/H_{e1}} \times P_{k/H_{e2}}) + [P_{k/H_p} + P_{k/H_f} - (P_{k/H_p} \times P_{k/H_f})] \times P_{k/H_{e1}} \times P_{k/H_{e2}} \quad (2.31)$$

If the assumption is made that the single hit cannot kill both engines (recall the assumption that only the component hit can be killed), then all of the component kills are mutually exclusive, and all of the products of the component kill probabilities in Equation 2.31 are zero. Hence, the aircraft is killed only if the pilot or the fuel tank is killed, and the $P_{K/H}$ and A_V are:

$$P_{K/H} = P_{k/H_p} + P_{k/H_f} \tag{2.32}$$

$$A_V = A_{v_p} + A_{v_f} \tag{2.33}$$

In general, only those components whose loss or damage can cause a kill of the aircraft on a single hit will contribute their vulnerable area to the total. If the single hit kills only one of the redundant components, the aircraft is not killed, and hence, nothing is contributed to the vulnerable area. Thus, the total vulnerable areas for this case is just the sum of vulnerable areas for each of the nonredundant critical components. Comparing Table 7 with Table 3 shows the single hit vulnerable area reduces from 52 ft² to 22 ft² due to the addition of the second engine. Thus, redundancy can significantly reduce the vulnerable area of the aircraft. On the other hand, if the damage to the redundant component which is hit creates secondary damage mechanisms or processes that propagate to another redundant component and kills that component, causing a loss of the aircraft, the redundant components will contribute to the aircraft vulnerable areas. For example, suppose the probability that a hit on one of the engines will cause that engine to throw blades into, or torch, or burn the other engine is 0.1. Because this can happen regardless of the engine hit, the component presented area becomes 50 + 50, or 100 ft², and the vulnerable area contributed by both engines

is 10 ft². Thus, this failure mode increases the aircraft vulnerable area to 32 ft².

(4) Aircraft Model Composed of Redundant Components With Overlap. If redundant components are now allowed to overlap one another, as shown by the aircraft in Figure 2.18, the computation of the vulnerable area given by Equation 2.29 must be modified because a single hit in the overlap region can kill both engines.

For this case, the cross hatched area shown in Figure 2.18 is defined as the overlap area. A single hit penetrating this area will have a probability of killing both redundant components, and hence the aircraft. Thus, it will be necessary to add the vulnerable area of the overlap region to that of the nonredundant critical components. In essence, the overlap region becomes another critical component, as in the nonredundant model with overlap. The vulnerable area is computed in the same manner as described previously; however, the details are slightly different. The expression for P_{s/h_o} given by Equation 2.20 must be modified. According to Equation 2.20, the probability that the aircraft survives a hit on an overlap area with no redundant components is given by:

$$P_{s/h_o} = P_{s_1} \times P_{s_2} \times P_{s_3} \times \ldots \times P_{s_N} \tag{2.34}$$

However, if there are two redundant components among the components along the shotline, such as components number 2 and number 3, the probability that both are killed, which is assumed to cause an aircraft kill, is equal to the product of their individual probabilities of kill, $(P_{k/h_2} \times P_{k/h_3})$. The probability that both components are not killed, which is required for aircraft survival, is the complement of $(P_{k/h_2} \times P_{k/h_3})$, or $[1-(P_{k/h_2} \times P_{k/h_3})]$. Thus, $(P_{s_2} \times P_{s_3})$ in Equation 2.34 must be replaced with $[1-(P_{k/h_2} \times P_{k/h_3})]$ and the result is given by Equation 2.35.

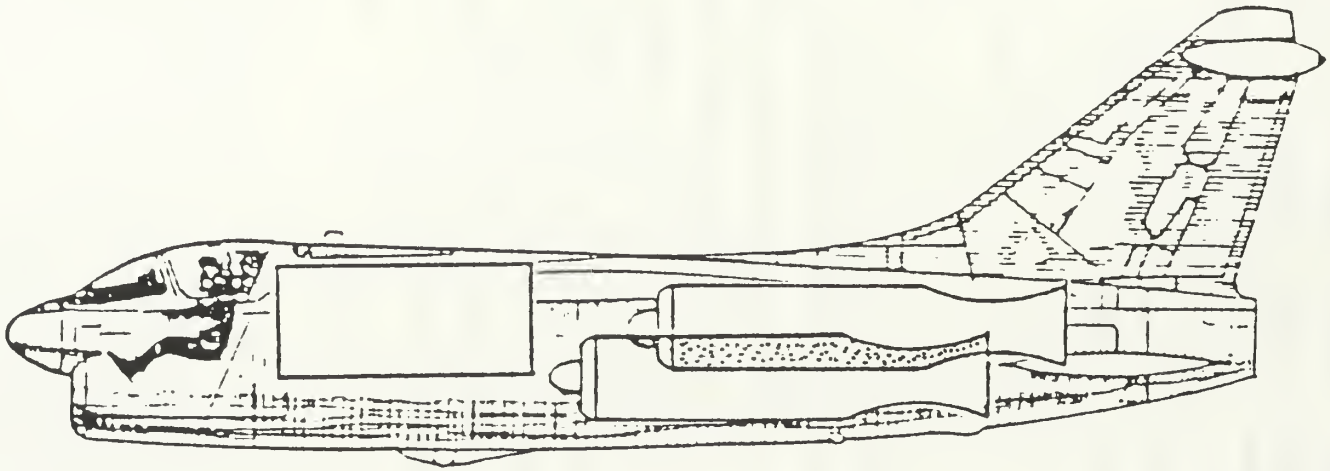


Figure 2.18 Redundant Aircraft Model With Overlap

$$P_{s/h_o} = P_{s_1} \times [1 - (P_{k/h_2} \times P_{k/h_3})] \times \ldots \times P_{s_N} \quad (2.35)$$

For our example, the probability the aircraft will survive a hit on the overlap region is given by:

$$P_{s/h_o} = 1 - (P_{s/h_{e1}} \times P_{s/h_{e2}}) \quad (2.36)$$

and the probability of kill given a hit on the overlap region is:

$$P_{k/h_o} = 1 - [1-(P_{k/h_{e1}} \times P_{k/h_{e2}})] \quad (2.37)$$

This procedure can be extended to the situation where there are three or more redundant overlapping components or multiple sets of overlapping redundant components.

The "elsewhere" or non-overlapping areas of each of the redundant components are not used in the vulnerable area computations for the same reason as that used in the no overlap case. A single shotline through any one of the redundant components outside of the overlap region causes only a kill of that component, not of the aircraft, and hence no contribution is made to the aircraft vulnerable area. If the $P_{k/h}$ values for the engines in the overlap region shown in Figure 2.18 are taken as 0.6 for the first engine hit and 0.2 for the overlapped engine (the overlapping engine slows the damage mechanism down), the probability the aircraft will survive a hit on the overlap region is given by Equation 2.36 and is equal to 0.88. The probability of an aircraft kill given a hit in the overlap region is given by Equation 2.37 and is equal to 0.12. If the overlap area is assumed to be 10 ft², the vulnerable area increases to 23.2 ft² due to the overlapping engines as shown by the computation in Table 8.

TABLE 8

Redundant Aircraft Model With Overlap

Critical Component	A_{P_i}	x	P_{k/h_i}	=	A_{v_i}	P_{k/H_i}
Pilot	4 ft²		1.0		4.0 ft²	.0133
Fuel	60 ft²		0.3		18.0 ft²	.0600
Overlap Area	10 ft²		0.12		1.2 ft²	.0040

A_P = 300 ft² A_V = 23.2 ft² $P_{K/H}$ = .0773

b. Multiple Hit Vulnerability

The analysis will now progress to the more reasonable expectation that in any combat engagement, an aircraft, if hit, will receive more than one hit. The distribution of these hits over the aircraft is assumed to be random, and all hits are assumed to travel along shotlines from the same direction. This latter assumption is not required, but is taken for ease of explanation.

The probability the ith component still survives after n random hits on the aircraft, denoted by $\bar{P}^{(n)}_{K/H_i}$, is equal to the product of the component survival probabilities for each of the n hits on the aircraft. The superbar notation on P indicates the joint probability, and the superscript n in parentheses indicates the hit number. Thus,

$$\bar{P}^{(n)}_{s_i} = P^{(1)}_{s/H_i} \times P^{(2)}_{s/H_i} \times \ldots \times P^{(n)}_{s/H_i} \qquad (2.38)$$

where P_{s/H_i} is the probability the ith component survives the jth hit on the aircraft. The probability of survival of

the ith component due to the jth hit on the aircraft is equal to one minus the probability of kill of the ith component due to the jth hit on the aircraft. Thus,

$$P_{s/H_i}^{(j)} = 1 - P_{k/H_i}^{(j)} \tag{2.39}$$

Recall that P_{k/H_i} is assumed to be a constant value for all j. Thus, Equation 2.38 can be given in the form:

$$P_{s/H} = \prod_{j=1}^{n} (1 - P_{k/H_i}^{(j)}) = [\ 1 - P_{k/H_i}\ ]^n \tag{2.40}$$

The probability of survival of the aircraft after n hits can be derived in a similar manner to give:

$$\bar{P}_{S/H}^{(n)} = \prod_{j=1}^{n} (\ 1 - P_{K/H}^{(j)}\ ) \tag{2.41}$$

where $P_{K/H}^{(j)}$ is the probability of kill of the aircraft due to the jth hit on the aircraft, and may or may not be constant for all j. The probability the aircraft is killed after n hits, $\bar{P}_{K/H}^{(n)}$, is the complement of $\bar{P}_{S/H}^{(n)}$, or:

$$P_{K/H}^{(n)} = 1 - P_{S/H}^{(n)} = 1 - \prod_{j=1}^{n} (1 - P_{K/H}^{(j)}) \tag{2.42}$$

In any multiple hit assessment, it is necessary to keep in mind the distinction between the effect of multiple hits on the vulnerable area of a nonredundant aircraft model as opposed to hits on a redundant aircraft model. Multiple hits on a nonredundant aircraft model do not change the total vulnerable area and the $P_{K/H}$ because of the assumption that components are either fully functional or killed. If a shot hits the aircraft, but not a critical component, the vulnerable area and the $P_{K/H}$ remain the same. Only when a hit actually strikes the vulnerable area of a nonredundant critical component is the aircraft killed.

The redundant aircraft model has to be viewed differently. If the redundant aircraft takes the first hit in the vulnerable area of a redundant component, the aircraft is not killed, but the aircraft vulnerable area and the $P_{K/H}$ will increase for the second hit because one of the redundant components has been killed. For instance, if one of two engines is killed on the first hit, the aircraft vulnerable area is now increased by the vulnerable area of the remaining engine because a kill of the remaining engine on a subsequent hit causes an aircraft kill.

Three methods are presented below to show the effects of multiple hits: The kill tree diagram, the state transition matrix (or Markov chain) method, and a simplified approach. The first method is more of an instructional tool, whereas the transition matrix method can be used in complex problems beyond the practical capability of the kill tree diagram. The simplified approach is the easiest to use.

(1) The Kill Tree Diagram. The sequence of events explained above regarding the effects of multiple hits can be illustrated diagrammatically using what is known as a kill tree diagram. The probability of kill of each component given a random hit on the aircraft is first computed using Equation 2.6, then the kill tree diagram is created. To simplify the explanation, consider the nonredundant aircraft model with no overlap illustrated in Figure 2.15 and defined in Table 3.

The Kill Tree Diagram, Nonredundant Model. Figure 2.19 presents the kill tree diagram that defines the mutually exclusive kill probabilities of each nonredundant critical component (pilot, fuel, and engine), and hence the aircraft, and the probability that no critical components are killed after the first hit on the aircraft.

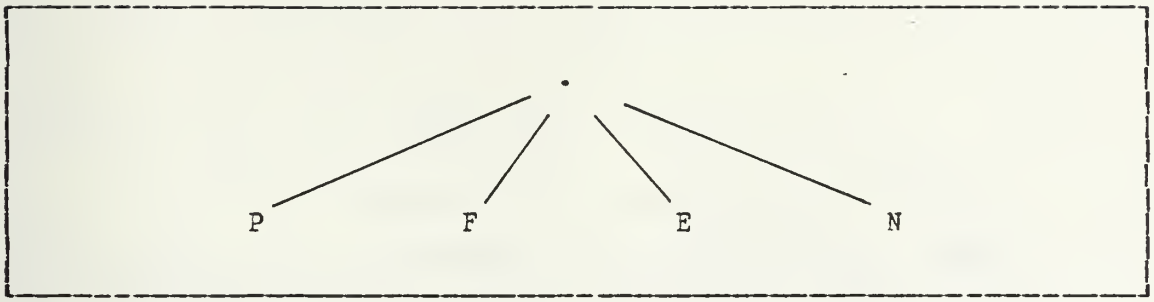


Figure 2.19 First Hit Nonredundant Kill Tree

In Figure 2.19, P = P_{k/H_p}, F = P_{k/H_f}, E = P_{k/H_e}, and N represents the probability that no critical components are killed and is given by N = 1-(P+F+E). Note that P+F+E+N is unity; all possibilities have been accounted for on this first hit. The probability the aircraft is killed on this first hit is given by P+F+E.

Figure 2.20 represents the kill tree diagram after the second hit. PxP represents the situation where the first hit killed the pilot, and the second hit also killed the pilot. It is important to note, however, that once a probability of kill is defined for each critical component on the first hit, that component is considered killed at that probability value for all subsequent hits. The pilot cannot be killed twice. The four branches from that kill probability for the second hit adds nothing new (no additional probability of pilot kill) to the sequence. This fact can be verified by examining the sum of the kill probabilities PxP PxF PxE PxN, which is the same as Px(P+F+E+N). Thus, this line is equal to the probability calculated for P on the first hit because P+F+E+N is unity. The only addition to the kill probability of the aircraft due to the second hit comes from critical components not killed on the first hit. This concept will become clearer and more important when redundant components are discussed.

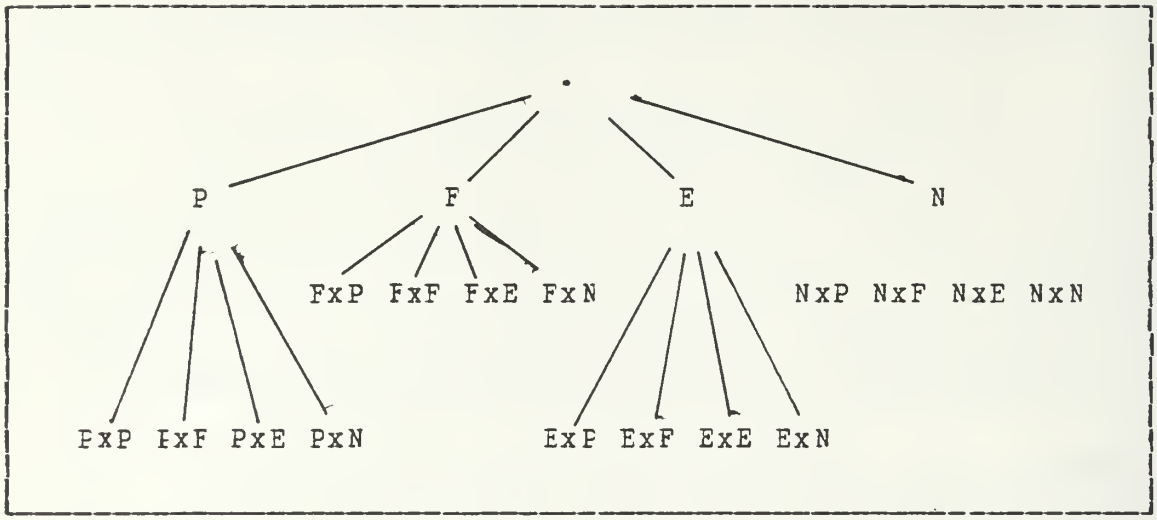


Figure 2.20 Second Hit Nonredundant Kill Tree

In order to illustrate the develcpment of a kill tree diagram, assume the numerical values fcr the component kill probabilities given in Table 3. Figure 2.21 illustrates the kill tree for the first hit. The

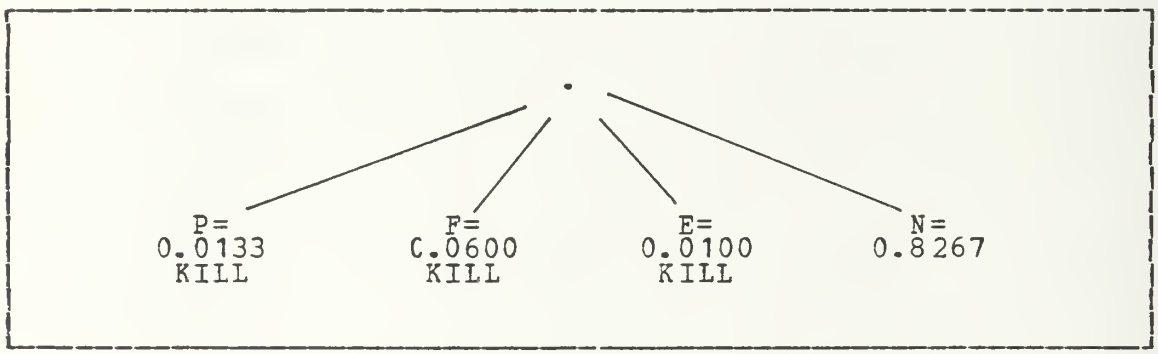


Figure 2.21 First Hit Nonredundant Kill Tree Example

probability the aircraft is killed after the first hit is the sum cf the kill probabilities for each of the critical components. Thus,

$$\tilde{P}_{K/H}^{(1)} = .0133 + .060C + .1000 = 0.1733 \qquad (2.43)$$

and hence,

$$\bar{P}_{S/H}^{(1)} = 1 - 0.1733 = 0.8267 \qquad (2.44)$$

Figure 2.22 extends this example to the second hit. The probability the aircraft is killed after the second hit is the sum of the additional kill probabilities for each of the critical components for the second hit. Thus,

$$\bar{P}_{K/H}^{(2)} = \bar{P}_{K/H}^{(1)} + [0.8267 \times (.0133 + .0600 + .1000)] \qquad (2.45)$$

and hence,

$$\bar{P}_{S/H}^{(2)} = 1 - \bar{P}_{K/H}^{(2)} = 1 - .3166 = .6834 \qquad (2.46)$$

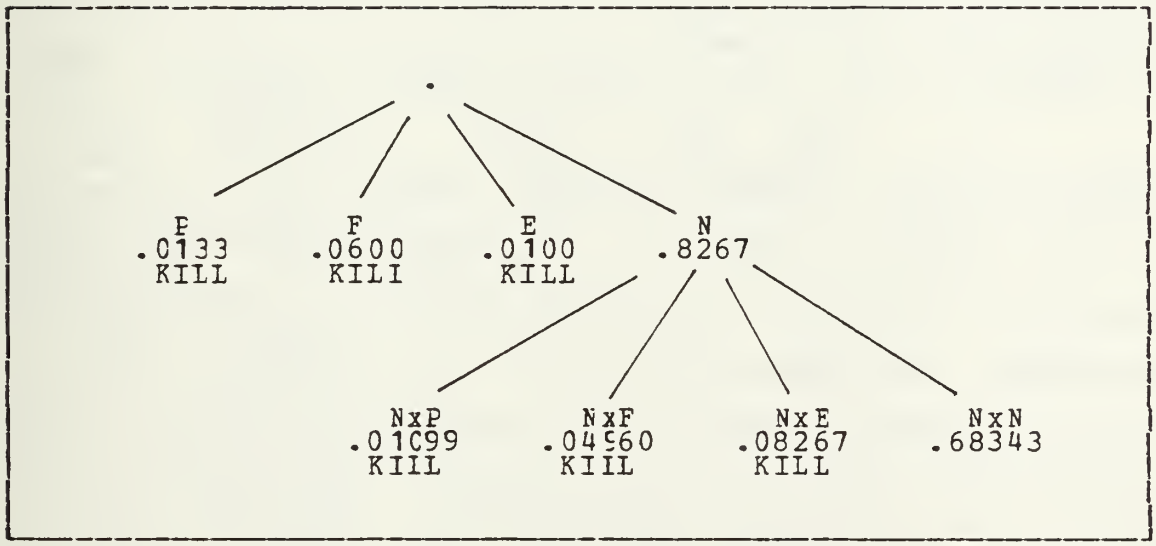


Figure 2.22 Second Hit Nonredundant Kill Tree Example

The kill tree diagram procedure may be continued indefinitely to determine $\bar{P}_{S/H}$ for any number of

hits. However, the probability the nonredundant aircraft model survives a sequence of hits can also be computed using Equation 2.41. For the nonredundant aircraft model, $P_{K/H}$ is constant as explained above. Thus, the probability the aircraft survives two hits is given by:

$$\bar{P}_{S/H}^{(2)} = (1 - P_{K/H}^{(1)}) \times (1 - P_{K/H}^{(2)}) = (1 - P_{K/H})^2 \qquad (2.47)$$

or,

$$\bar{P}_{S/H}^{(2)} = (1 - 0.1733)^2 = 0.6834 \qquad (2.48)$$

Note that this value is the same as that obtained from the kill tree diagram, as it should be.

Equation 2.41 can be used for any number of hits and is much easier to use than the kill tree diagram. The essence of this equation is that all of the nonredundant critical components can be combined into one composite critical component whose vulnerable area is 52 ft² and whose $P_{K/H}$ is 0.1733 in the numerical example.

Kill Tree Diagram, Redundant Model. Consider now the redundant aircraft model shown in Figure 2.17 and defined in Table 7. An evaluation for $\bar{P}_{K/H}^{(n)}$ and $\bar{P}_{S/H}^{(n)}$ can be performed in a manner similar to the previous discussion. Although the engines are redundant critical components, each must be shown as a separate branch in the kill tree diagram, because a kill of an engine is a possible outcome of an aircraft hit; and any engine kill will have an effect on the aircraft's vulnerability. Figure 2.23 illustrates the kill tree diagram for the first hit. Note that N now represents the probability that no nonredundant or redundant component is killed.

The logical kill expression for this redundant aircraft model is given by Equation 2.49.

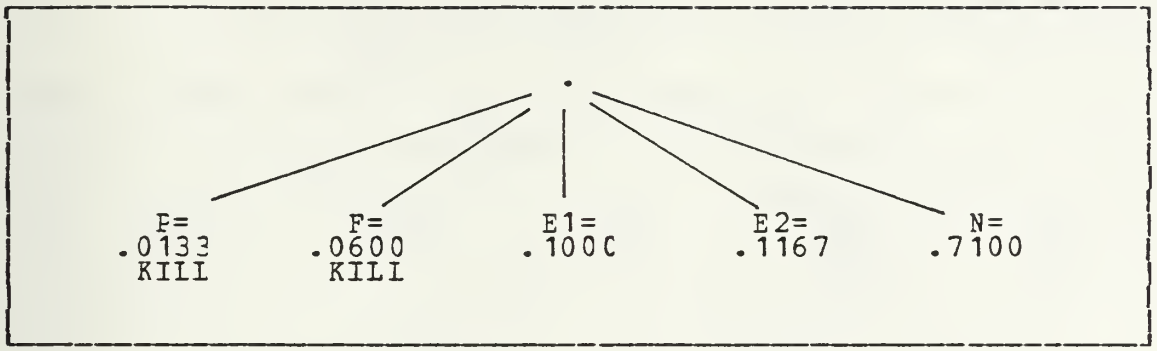


Figure 2.23 First Hit Redundant Kill Tree Example

(Pilot) .OR. (Fuel Tank) .OR. (2.49)
[(Engine 1) .AND. (Engine 2)]

Because the first hit cannot kill both engines, the probability that the aircraft is killed after the first hit is just the sum of the kill probabilities for each of the nonredundant critical components (pilot and fuel). Thus,

$$\bar{P}_{K/H}^{(1)} = 0.0133 + 0.0600 = 0.0733 \qquad (2.50)$$

Figure 2.24 illustrates the event probabilities on the second hit after a kill of engine 1 on the first hit. The sequence represented by killing engine 1 on the first hit and then killing the pilot (0.00133), or fuel (0.0600), or engine 2 (0.01167) on the second hit results in additional aircraft kills. Thus, the cumulative probability of an aircraft kill is due to kills of the nonredundant critical components as well as kills defined by component redundancy restrictions. The five branches from a kill of engine 2 and from the N branch will also contribute additional kills. Thus, after two hits, the cumulative probability of kill is given by Equation 2.51.

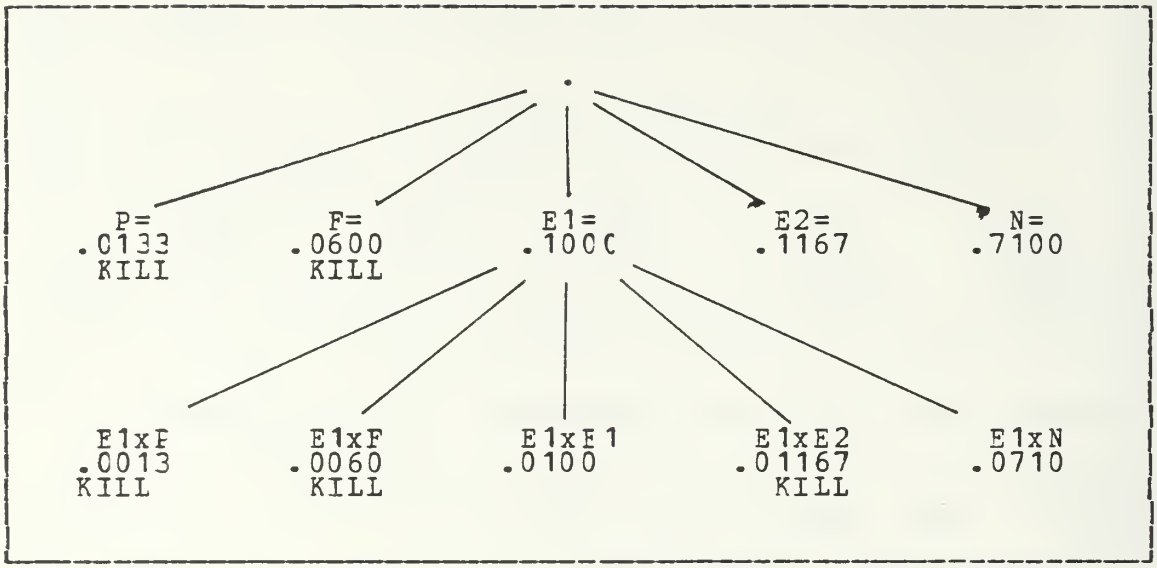


Figure 2.24 Second Hit After First Hit on Engine 1

$$\bar{P}^{(2)}_{K/H} = .0733 + [.1000 \times (.0133 + .0600 + .1167)] + [.1167 \times (.0133 + .0600 + .1000)] + [.7100 \times (.0133 + .0600)] = 0.1646 \quad (2.51)$$

and hence:

$$\bar{P}^{(2)}_{S/H} = 1 - 0.1646 = 0.8354 \quad (2.52)$$

Note the significant increase in survivability (0.8354 versus 0.6834) after the second hit due to the addition of the second engine.

This procedure can be continued indefinitely, as in the nonredundant case, but it is obvious that the computations quickly become overwhelming in complexity. The state transition matrix method described below is better suited to handle the problem.

(2) The State Transition Matrix Method (Markov Chain). Briefly, the state transition matrix method assumes that a sequence of independent events (random hits on the aircraft) can be modelled as a Markov process. In a Markov process, the aircraft is defined to have two or more states in which it may reside, and the probability of an aircraft kill due to the j+1 hit is the probability that the j+1 hit on the aircraft will cause the aircraft to transition from a non-kill state after j hits to a kill state. The sequential process of evaluating the probability the aircraft exists in each of the several possible states after hits 1, 2, 3, ..., J is based upon the probability the aircraft existed in each of the possible states after hits 0, 1, 2, ..., J-1, respectively, and is referred to as a Markov chain. Rather than dwell on the mathematical theory, an example using the previously defined redundant aircraft model will serve much better to illustrate the methodology.

An aircraft consisting of a pilot, a fuel tank, and two engines can exist in five distinct states:

1. One or more of the nonredundant critical components (the pilot and the fuel tank) have been killed, resulting in an aircraft kill, denoted by Knrc.
2. Only engine 1 has been killed, denoted by Krc1.
3. Only engine 2 has been killed, denoted by Krc2.
4. Both engine 1 and engine 2 have been killed, resulting in an aircraft kill, denoted by Krc.
5. None of the nonredundant critical components and neither of the engines are killed, denoted by NK.

States Knrc and Krc are called absorbing states because the aircraft cannot transition from these two kill states to any of the other three non-kill states.

A transition matrix of probabilities, [T], can now be constructed to specify how the aircraft will transition from one state to another due to a hit on the aircraft. Table 9 illustrates the computation of the [T] matrix for the example redundant aircraft model defined in Table 7. Each element of the matrix represents the

TABLE 9

Computation of the State Transition Matrix

Probability of transitioning from this state

	Knrc	Krc1	Krc2	Krc	NK	to this state
	300	4+18	4+18	0	4+18	Knrc
1/300	0	30+213	0	0	30	Krc1
	0	0	35+213	0	35	Krc2
	0	35	30	300	0	Krc
	0	0	0	0	213	NK

Note that the sum of each column is unity

probability of transitioning from the state defined by the column locations to the new state defined by the row location. The matrix is read as follows. The probability of the aircraft transitioning from the Knrc state to the Knrc state is unity (300/300) because Knrc is an absorbing state. The probability of transitioning from the Krc1 state (kill of engine 1) to the Knrc state (kill of a nonredundant component) is the sum of the conditional probabilities of kill of the two nonredundant components, that is, P+F. The probability of transitioning from Krc1 to Krc1 (remaining in

Krc1) is the sum of engine 1's probability of kill given a hit on the aircraft, E1, and that of the remaining "elsewhere" area of the aircraft, N. Transitioning from Krc1 to Krc2 is zero because a kill of the second engine after the first engine is killed defines the state Krc. Thus, the state transitions from Krc1 to Krc according to the conditional probability of kill of the second engine, E2, and so on.

Let the probability that the aircraft exists in each of the five possible states after the jth hit be expressed by a vector $[S]^{(j)}$, where

$$[S]^{(j)} = \begin{bmatrix} Knrc \\ Krc1 \\ Krc2 \\ Krc \\ NK \end{bmatrix}$$

Note that the sum of the elements in $[S]^{(j)}$ is always unity; the aircraft must exist in one of these five states. The probability the aircraft is in each one of the five states after the (j+1)th hit is given by:

$$[S]^{(j+1)} = [T][S]^{(j)} \tag{2.53}$$

That is, the aircraft transitions from $[S]^{(j)}$ to $[S]^{(j+1)}$ according to [T].

An aircraft kill is defined by those states that specify either a kill of any of the nonredundant components or a kill of enough members of the sets of redundant components, such as both engines. In this example, Knrc and Krc specify the kill states. Hence, the probability the aircraft is killed after n hits is given by:

$$\bar{P}_{K/H}^{(n)} = Knrc^{(n)} + Krc^{(n)} \tag{2.54}$$

where $Knrc^{(n)}$ and $Krc^{(n)}$ are the probabilities the aircraft is in those two states after n hits.

Using the numbers generated in the previous numerical example, consider the first hit. Prior to the first hit, the aircraft is entirely in the NK state. Thus, according to Equation 2.53:

$$[S]^{(1)} = [T][S]^{(0)} = [T] \begin{bmatrix} C \\ C \\ C \\ C \\ 1 \end{bmatrix} \quad NK$$

Carrying out the matrix multiplication gives:

$$[S]^{(1)} = \begin{bmatrix} 0.0773 \\ 0.1000 \\ 0.1167 \\ 0 \\ 0.7100 \end{bmatrix} \quad \begin{matrix} Knrc \\ \\ \\ Krc \\ \end{matrix}$$

Thus, $\bar{P}_{K/H}^{(1)} = 0.0733$ as before. Similarly, for the second hit:

$$[S]^{(2)} = [T][S]^{(1)} = [T] \begin{bmatrix} 0.0733 \\ 0.1000 \\ 0.1167 \\ 0 \\ 0.7100 \end{bmatrix}$$

Carrying out the matrix multiplication gives:

$$[S]^{(2)} = \begin{bmatrix} 0.1413 \\ 0.1520 \\ 0.1793 \\ 0.0233 \\ 0.5401 \end{bmatrix} \quad \begin{matrix} Knrc \\ \\ \\ \\ Krc \end{matrix}$$

Note that the sum of the elements of [S] is unity, as it should be. The $[S]^{(2)}$ vector results reveal that after the second hit there is a 14.13 percent probability that either the pilot or the fuel tank or both have been killed, a 15.20

percent probability that engine 1 has been killed, a 17.93 percent probability that engine 2 has been killed, and a 50.41 percent probability that none of the critical components have been killed. Thus, after the second hit:

$$\bar{P}_{K/H}^{(2)} = 0.1413 + 0.0233 = 0.1646 \qquad (2.55)$$

This value is the same as that obtained from the kill tree diagram after the second hit, as it should be. This process can easily be continued for as many hits as desired. Figure 2.25 shows the $\bar{P}_{K/H}^{(n)}$ as a function of n for both the redundant aircraft model and the nonredundant aircraft model given in Table 3. The difference between the two curves is the reduction in vulnerability due to redundancy.

In the above presentation, the transition matrix was assumed to be the same for all hits. This assumption is not necessary. If multiple damage mechanisms hit the aircraft from several different aspects, a transition matrix can be constructed for each aspect of interest. The computation of the state vector for the j+1 hit, given by Equation 2.53, would use the transition matrix for the approach direction of that particular hit. Another possible modification is the consideration of an increase in P_{K/H_i} due to multiple hits. Again, [T] could be changed from one hit to the next.

(3) A Simplified Approach. If the probability of survival of each of the critical components after n hits on the aircraft is known, an approximation for the probability the aircraft has been killed by the n hits can be obtained by neglecting the mutually exclusive feature of the individual component kills on any one hit. Thus, for the example redundant component aircraft model, Equation 2.30 can be used. Equation 2.30, for the n hit situation, is given by Equation 2.56.

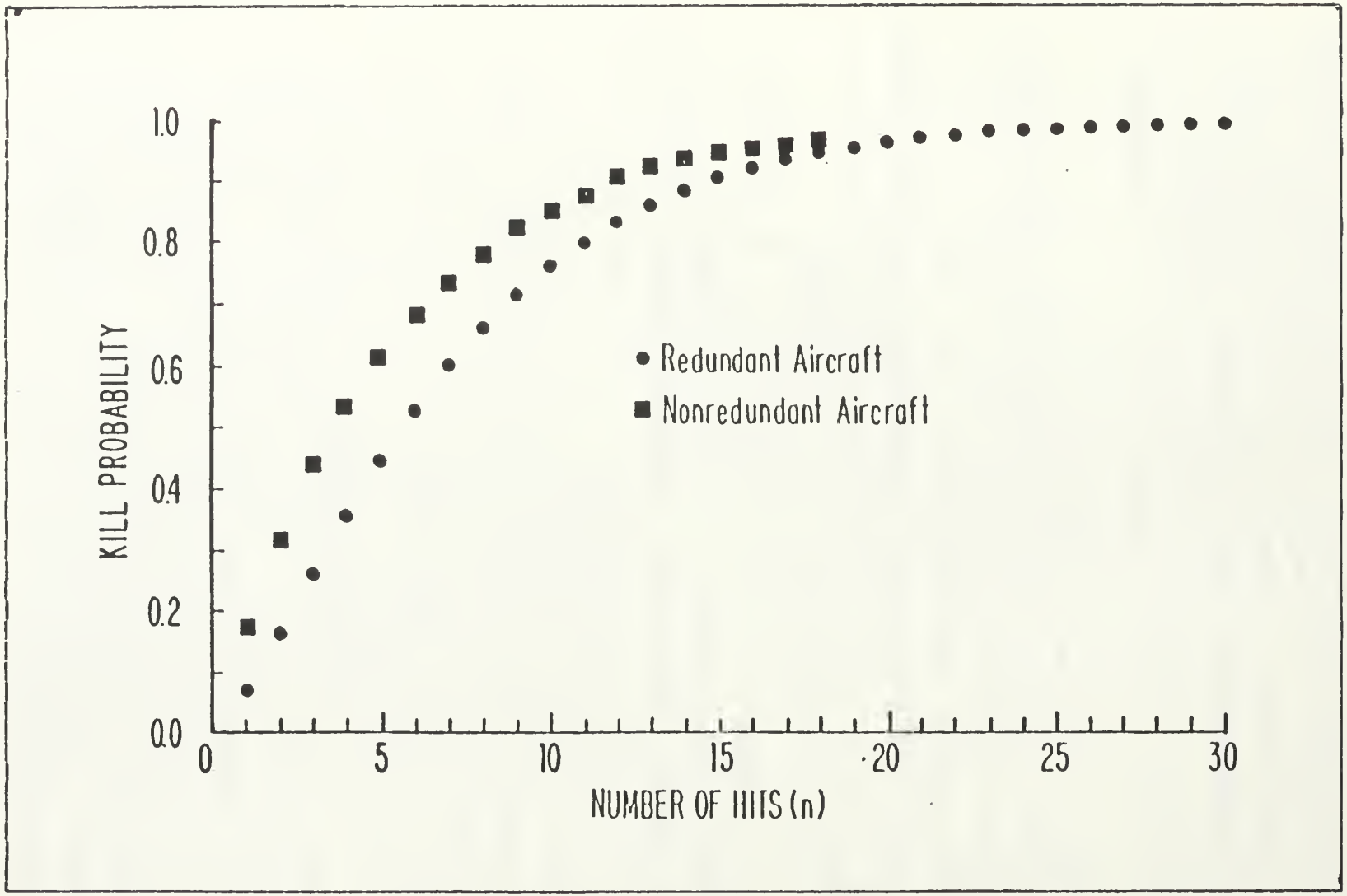


Figure 2.25 Probability of Kill for the Redundant Aircraft Model vs the Number of Hits

$$\bar{P}^{(n)}_{S/H} = (1-\bar{P}^{(n)}_{k/H_p})(1-\bar{P}^{(n)}_{k/H_f})(1-(\bar{P}^{(n)}_{k/H_{e1}})(\bar{P}^{(n)}_{k/H_{e2}})) \quad (2.56)$$

where

$$\bar{P}_{k/H_p} = 1 - (1-P_{k/H_p})^n \ , \ \bar{P}_{k/H_f} = 1 - (1-P_{k/H_f})^n \quad (2.57)$$

and

$$(\bar{P}^{(n)}_{k/H_{e1}})(\bar{P}^{(n)}_{k/H_{e2}}) = (1-(1-P_{k/H_{e1}})^n)(1-(1-P_{k/H_{e2}})^n) \quad (2.58)$$

according to Equation 2.40. Table 10 presents the $\bar{P}^{(n)}_{K/H}$ for both the transition matrix method and the simplified approach for several values of n. Note that the approximate $\bar{P}^{(n)}_{K/H}$ is both lower than and higher than the correct answer and that the approximate kill probability is reasonably close to the correct value, for this example.

TABLE 10

A Comparison of Aircraft Kill Probabilities

Hits, n	1	3	5	10	20
$\bar{P}^{(n)}_{K/H}$ Correct	.0733	.2615	.4456	.7619	.9640
$\bar{P}^{(n)}_{K/H}$ Approximate	.0833	.2693	.4436	.7470	.9567

(4) Multiple Hit Vulnerable Area. The cumulative probability of kill given n hits derived above is not necessarily the best measure for assessing or comparing

aircraft designs due to its dependence on the physical size of the aircraft. If two aircraft have identical vulnerable areas, but different presented areas, the one with the largest presented area will appear to be less vulnerable because its cumulative probability of kill given n hits will be less than that of the aircraft with the smaller presented area. On the other hand, being larger, it may suffer more hits; that is, it may be more susceptible.

The measure that is the most meaningful for vulnerability assessment and comparison of designs is vulnerable area. For nonredundant aircraft, the probability of kill given a hit and the vulnerable area are constant for each and every hit. Each subsequent hit has just as much chance of killing the aircraft as the previous hit (assuming component degradation is neglected). However, this is not true for aircraft with redundant critical components. For these aircraft, the probability of kill given a hit and the corresponding vulnerable area changes with each hit because of the increasing possibility of the loss of one or more of the redundant components. In order to compute the multiple hit vulnerable area, an event-based probability of kill given a hit must be computed for each hit. In general, the probability of aircraft survival after taking n hits was given by Equation 2.41 which is:

$$\bar{P}_{S/H}^{(n)} = (1-P_{K/H}^{(1)})(1-P_{K/H}^{(2)}) \cdots (1-P_{K/H}^{(n)}) \tag{2.59}$$

which also can be expressed in the form:

$$\bar{P}_{S/H}^{(n)} = (\bar{P}_{S/H}^{(n-1)})\ (1-\bar{P}_{K/H}^{(n)}) \tag{2.60}$$

The value desired in Equation 2.60 is $\bar{P}_{K/H}^{(n)}$, the event-based probability that the aircraft is killed on the nth hit on the aircraft given that it has survived the first (n-1) hits. Rearranging terms in Equation 2.60 gives:

$$1 - P_{K/H}^{(n)} = (\bar{P}_{S/H}^{(n)}) \; / \; (\bar{P}_{S/H}^{(n-1)}) \tag{2.61}$$

Equation 2.61 can also be given in the form:

$$1 - P_{K/H}^{(n)} = (1 - \bar{P}_{K/H}^{(n)}) \; / \; (1 - \bar{P}_{K/H}^{(n-1)}) \tag{2.62}$$

and hence,

$$P_{K/H}^{(n)} = (\bar{P}_{K/H}^{(n)} - \bar{P}_{K/H}^{(n-1)}) \; / \; (1 - \bar{P}_{K/H}^{(n-1)}) \tag{2.63}$$

The vulnerable area for the nth hit, $A_V^{(n)}$, is computed using the $\bar{P}_{K/H}^{(n)}$ given by Equation 2.63 and the basic vulnerable area equation, Equation 2.1. Thus,

$$A_V^{(n)} = (A_P) \; (\bar{P}_{K/H}^{(n)}) \tag{2.64}$$

Figure 2.26 shows the $A_V^{(n)}$ for the redundant model $\bar{P}_{K/H}^{(n)}$ given in Figure 2.25. Note that the $A_V^{(1)}$ is just the sum of the vulnerable areas of the nonredundant components. Note also the asymptotic behavior for the redundant model. The constant vulnerable area of the nonredundant aircraft given in Table 3 is also plotted in Figure 2.26 for the purpose of comparison. Note that the vulnerable area of the redundant aircraft is less than that of the nonredundant aircraft (with the 30 ft² vulnerable area engine) for the first fifteen hits. On subsequent hits, the vulnerable area is slightly larger due to the fact that there is a strong likelihood that one or the other of the two engines has been killed, and the benefits of redundancy have been eliminated.

c. Presentation of Results

Nonexplosive projectile results normally will be presented in summary forms of total A_V of nonredundant

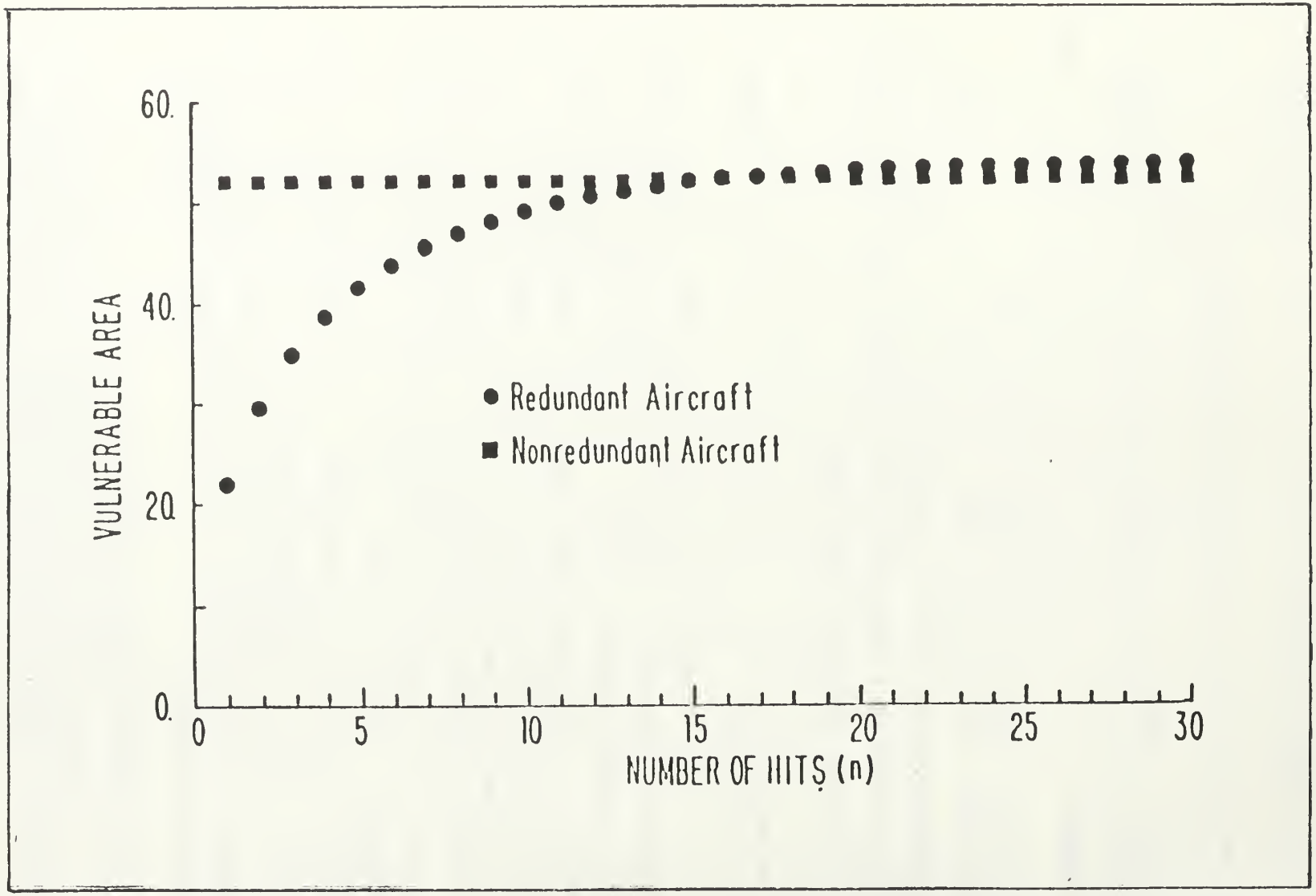


Figure 2.26 Vulnerable Area vs Number of Hits

components for each combination of threat, kill category, and striking aspect assessed with the striking velocity varying from 500 to 3500 ft/sec. Manual assessments for nonexplosive projectiles will be performed for at least the six major views of the aircraft. A computerized assessment will be performed for at least the six major views and usually for a total of 26 views spread at 45 degree increments of elevation and azimuth as described earlier. A typical total aircraft single hit vulnerable area summary form is shown in Figure 2.27. Multiple hit vulnerable area curves similar to the one shown in Figure 2.26 should also be presented for at least six aspects.

In addition to the total aircraft A_V presentation, the vulnerable area of each critical component should also be listed, and both the true and the incremental vulnerable areas should be presented for overlapping components. Redundant components should be identified, and the number of redundant components that must be killed to cause an aircraft kill should be noted. The single hit vulnerable area associated with overlapping redundant components should also be identified. Figure 2.28 shows a sample component vulnerable area summary form.

For explosive projectiles and contact fuzed missiles, vulnerability data normally will be presented as summary forms of the total A_V of nonredundant components for each combination of threat, kill category, and aspect angle assessed. These results usually are not presented as varying with the fragment striking velocity. Each major redundant component will be shown separately if assessed. Assessments will be performed for at least the six major views and for 26 views if possible. For HE projectiles, in addition to the total aircraft vulnerable area, the contribution by subsystem or aircraft region should also be presented. This can be done similar to the nonexplosive

Assessment date _____ Aircraft _____

Performing organization _____ Threat _____

Kill category _____

Projectile V_S, ft/sec (m/sec)	Total A_V, ft^2 (m^2)					
	Left side	Right side	Top	Bottom	Front	Rear
500 (152.4) 1,000 (304.8) 1,500 (457.2) 2,000 (609.6) 2,500 (762.0) 3,000 (914.4) 3,500 (1066.8)						

Figure 2.27 Total Aircraft Single Hit Vulnerable Form

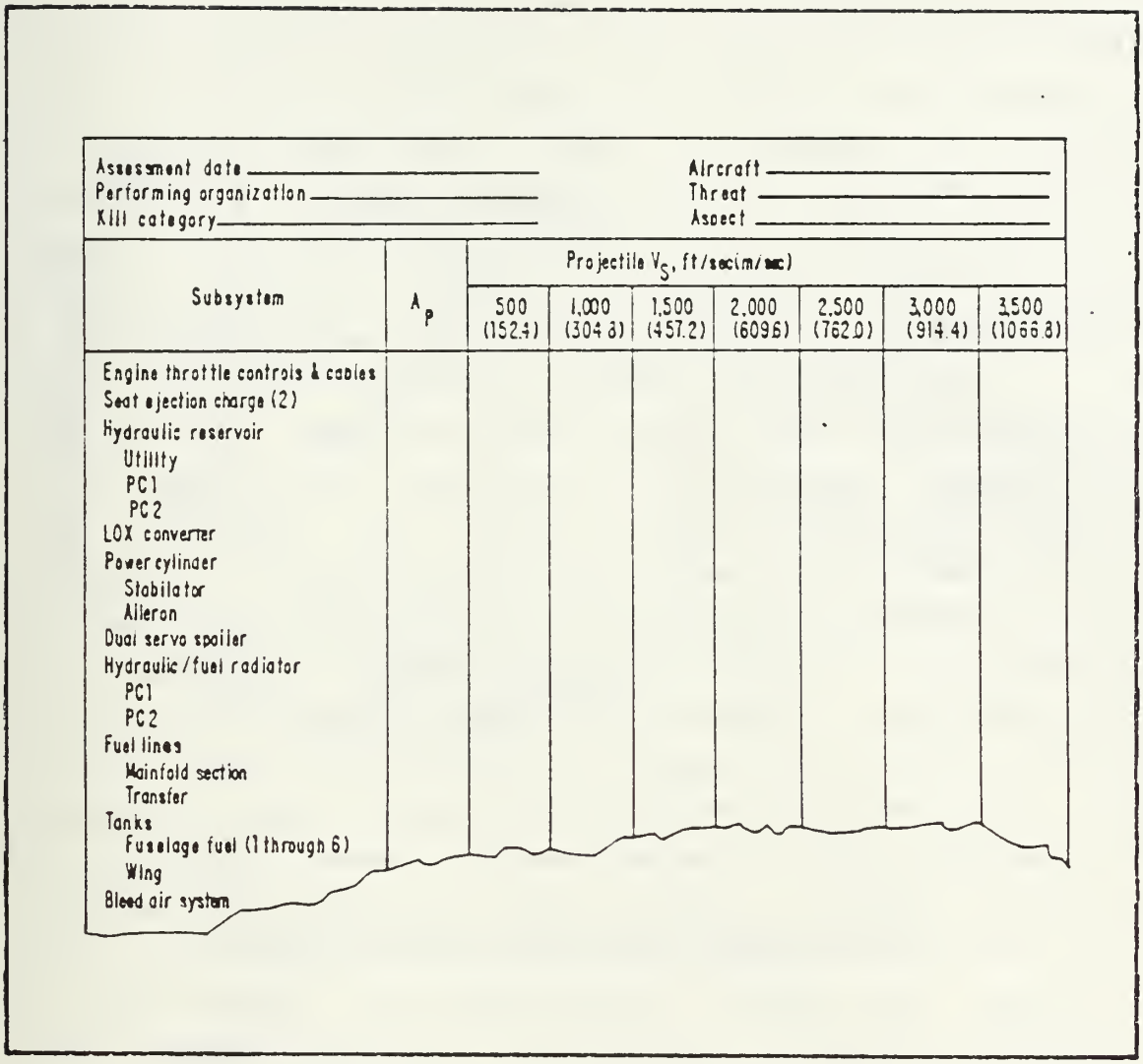


Figure 2.28 Sample Component Vulnerable Area Form

projectile, or in a sketch of the aircraft with regions shown and the region's presented area and vulnerable area summarized on an accompanying form.

2. Blast

Aircraft vulnerability to external blast is usually expressed as an envelope about the aircraft where the detonation of a specified charge weight of spherical uncased

pentolite high explosive will result in a specified level of damage or kill to the aircraft. Detonation outside of such an envelope will result in little or no damage to the aircraft or in a lesser kill level. The damage mechanism is the blast resulting from the detonation of the high explosive in the vicinity of the aircraft. A spectrum of charge weights are often specified for which the aircraft vulnerability measures are computed in the vulnerability assessment. The specific charge weights selected are representative of the expected threat warheads which might be encountered. Envelopes are determined for a variety of encounter conditions which account for variations in aircraft speed and altitude, as well as aspect. Aircraft critical components vulnerable to the external blast consist principally of portions of the airframe structure and control surfaces. Threshold kill criteria for the critical components are derived from structural and aerodynamic analyses. Once the blast pressures and impulse levels required for a component kill are determined for several locations on the aircraft surface, a contour may be plotted corresponding to the detonation distance and the weight of pentolite which will provide the required overpressure and impulse level. Two different graphical presentations of the data may be used. The first is a plot of charge weight versus distance for a constant kill level. Several curves can be drawn on the same graph, one for each altitude of interest. A similar graph is required at each azimuth and elevation angle of interest about the aircraft. Figure 2.29 is an example of this type of presentation. The second graphical method, illustrated in Figure 2.30, is to construct isocharge weight contours about the aircraft for a given kill level and altitude in all planes of interest.

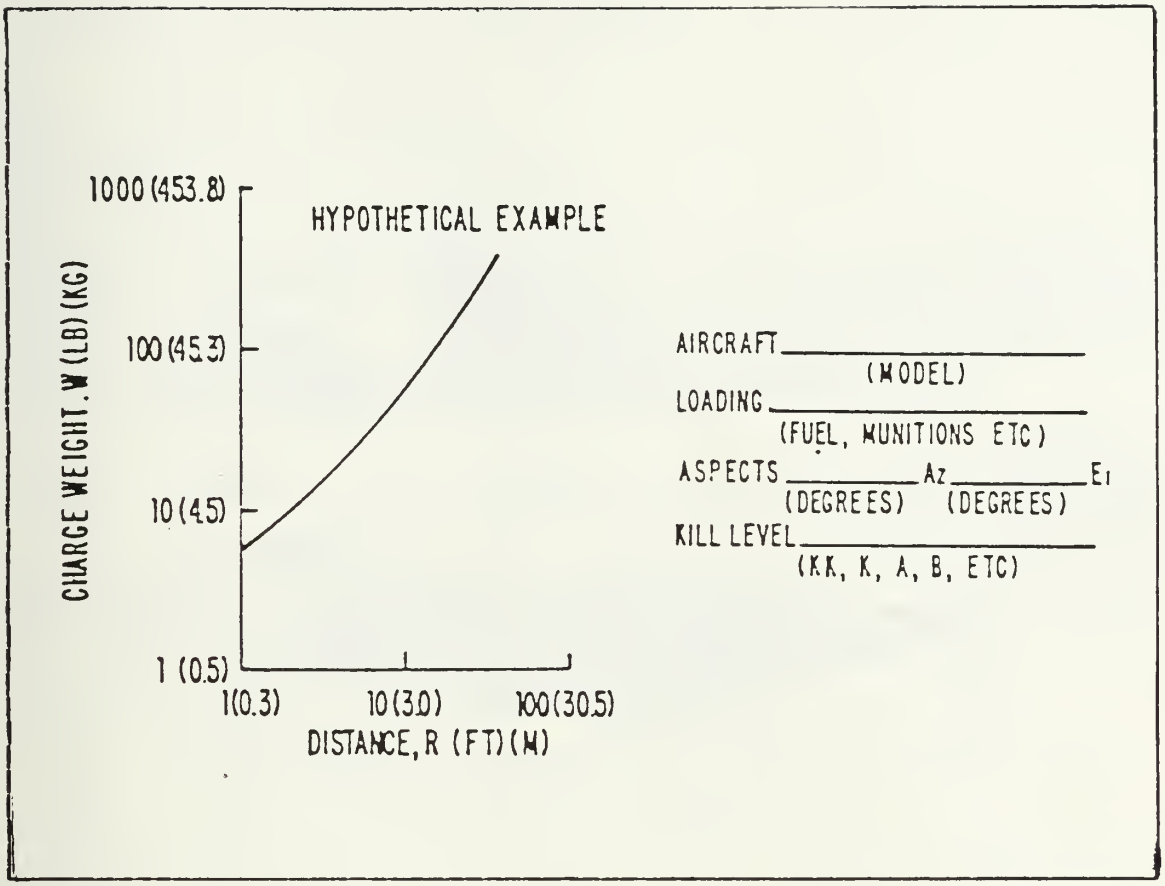


Figure 2.29 Typical External Blast Vulnerability Data Presentation

3. Endgame Analysis

The probability of an aircraft kill due to the burst of a specific warhead for a particular set of encounter conditions, $P_{K/D}$, is dependent upon how many fragments hit the aircraft and the aircraft's vulnerability to the multiple hits. The number of fragments which strike the aircraft was derived in the previous chapter and the aircraft's vulnerability to multiple hits was discussed in the previous section. The $P_{K/D}$ due to the n hits on the aircraft is analogous to the $\bar{P}_{K/H}^{(n)}$ derived in this chapter earlier. Thus, $P_{K/D}$ can be estimated using cumulative

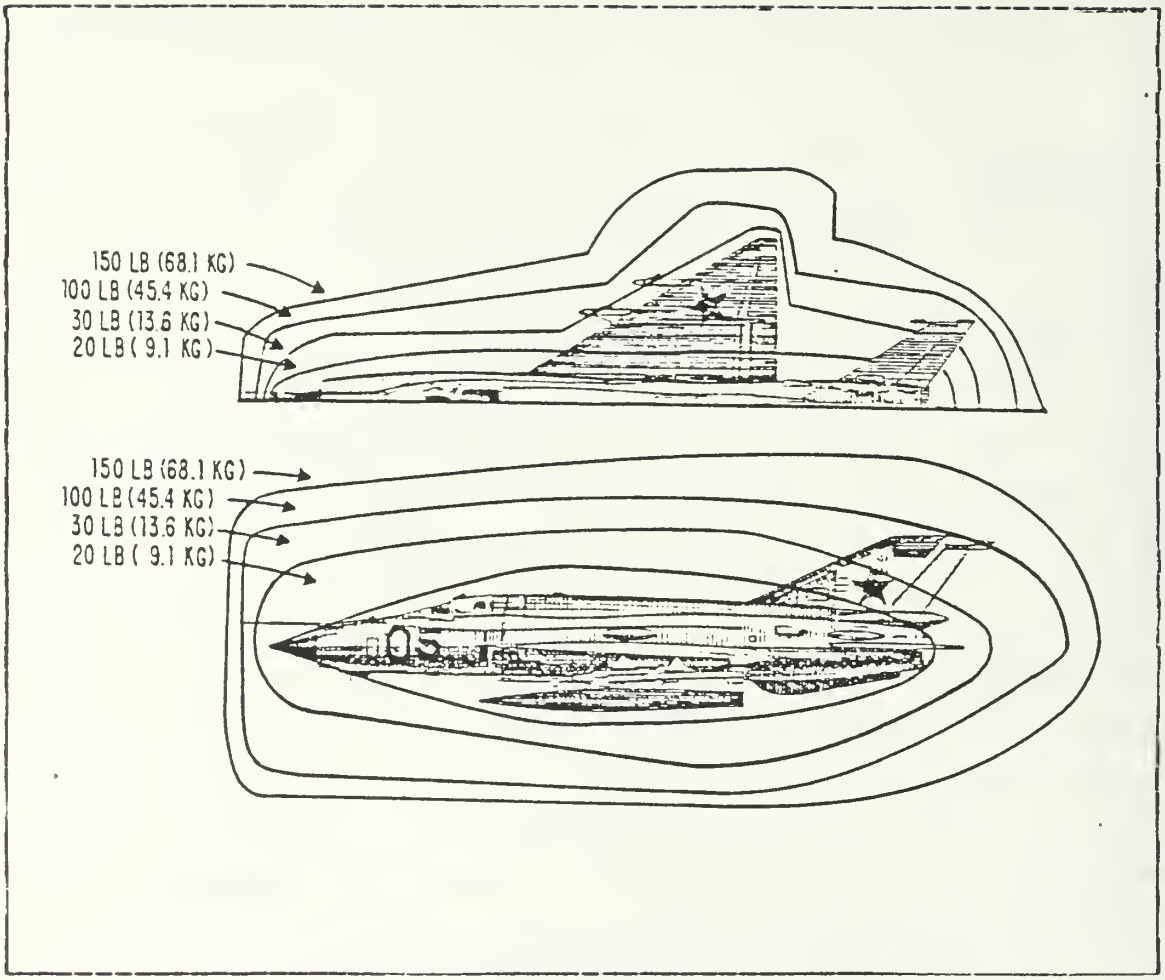


Figure 2.30 Typical External Blast K-Kill Contour for Various Weights of Uncased Pentolite at Sea Level

probability of kill curves similar to the one shown in Figure 2.25. Simplified equations for $P_{K/D}$ in terms of the aircraft vulnerable area and the n hits are derived below.

The probability the aircraft is killed given the jth random hit by a single fragment, $\bar{P}_{K/H}^{(n)}$, can be determined using the procedure described in this chapter. The probability that the aircraft is killed by the n independent, random hits from detonation, $\bar{P}_{K/H}^{(n)}$, is given by Equation 2.42. Therefore,

$$\bar{P}_{K/H}^{(n)} = P_{K/D} = 1 - \prod_{j=1}^{n} (1 - P_{K/H}^{(j)}) \qquad (2.65)$$

It can be shown that for small $P_{K/H}^{(j)}$,

$$\prod_{j=1}^{n} (1 - P_{K/H}^{(j)}) = \exp(- \sum_{j=1}^{n} P_{K/H}^{(j)}) \qquad (2.66)$$

Furthermore,

$$\sum_{j=1}^{n} P_{K/H}^{(j)} = \sum_{j=1}^{n} A_V^{(j)} / A_P \qquad (2.67)$$

Hence, $P_{K/D}$ can be given in the form:

$$P_{K/D} = 1 - \exp((- \rho / n) \sum_{j=1}^{n} A_V^{(j)}) \qquad (2.68)$$

according to Equation 1.33. If there are no redundant critical components, $A_V^{(j)}$ is usually assumed to be a constant value for all hits, and $P_{K/D}$ simplifies to:

$$P_{K/D} = 1 - \exp(- \rho A_V) \qquad (2.69)$$

An example of the computation of $P_{K/D}$ for an encounter is given in Table 11.

There is no unique value for $P_{K/D}$ for a warhead detonation at a specific location with respect to the aircraft. $P_{K/D}$ will be different for detonations at the same distance, but at different locations around the aircraft. Nevertheless, the aircraft's vulnerability to an externally detonating warhead is often indicated only with respect to the distance of the detonation from the aircraft, without regard to the other variables.

A typical curve relating $P_{K/D}$ to the detonation distance, R, is given in Figure 2.31. This curve is referred to as the kill function given a detonation, and the radius at which $P_{K/D}$ is equal to 0.5 is called the lethal radius of the warhead. The value of $P_{K/D}$ specified for each value of R could be the average of the $P_{K/D}$'s computed for

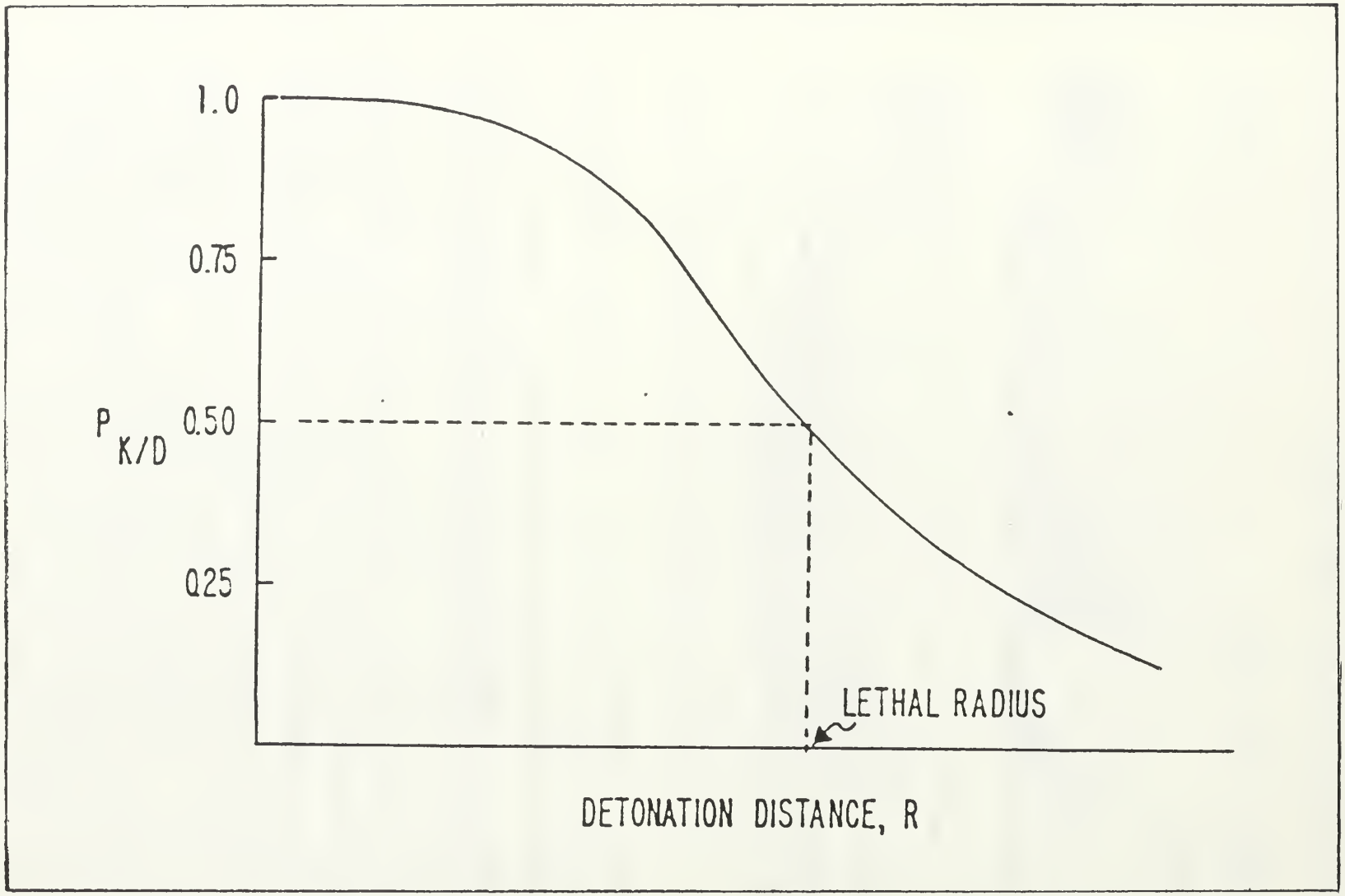


Figure 2.31 Typical Probability of Kill Given A Detonation Function

TABLE 11

An Example Computation for $P_{K/D}$

Static Warhead Parameters	Spray angles, $\alpha_1 = 50$, $\alpha_2 = 120$
	Number of fragments, N = 1000
	Fragment velocity, V_i = 7000 fps
Encounter Parameters	Missile speed, V_m = 1500 fps
	Missile angle, $\theta = 30$
	Detonation distance, R = 80 ft
	Aircraft speed, V_t = 1000 fps
Aircraft Parameters	Aircraft presented length = 50 ft
	Aspect vulnerable area, $A_V = 25$ ft^2 (to fragment size and striking velocity under consideration)
Fragmentation Dynamic Spray Angles	$\phi_1 = \text{Tan}^{-1}[7644 / 1515] - 30$
	$\phi_2 = \text{Tan}^{-1}[4250 / -5763] - 30$
Fragment Spray Density	$\rho = 1000 / [2 \times \quad \times 80^2 \times 1.059]$
	$\rho = .0235$ fragments/ft^2
Probability of kill	$P_{K/D} = 1 - \exp(-0.0235 \times 25)$
	$P_{K/D} = 0.44$

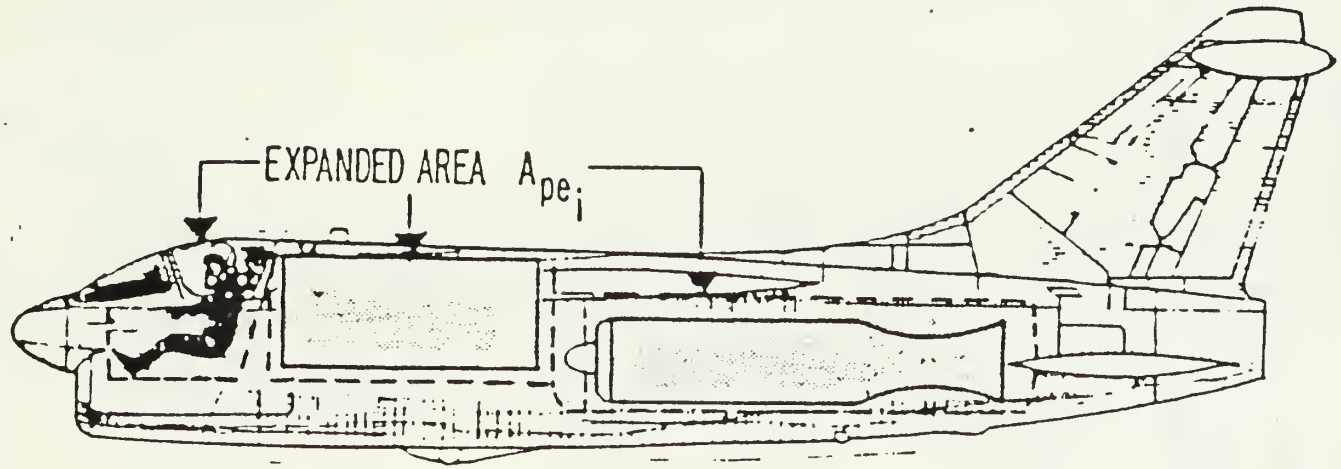
several different encounters at R, or the $P_{K/D}$ values could be weighted with respect to the expected probability of encounter occurrence in order to obtain a weighted average. For example, if a certain missile only approaches the aircraft from the rear aspect, only $P_{K/D}$ values for that type of encounter would be computed.

E. VULNERABILITY TO INTERNALLY DETONATING HE WARHEADS

Anti-aircraft projectiles 20mm and larger often have an HE core with a contact fuze that detonates the warhead either immediately or shortly after impacting the aircraft. This results in a detonation on or inside the aircraft, with the accompanying blast and fragment spray in many directions. The assumption of parallel trajectories or shotlines through the aircraft used in the nonexplosive penetrator vulnerability assessment is not valid in this situation. Instead, the fragment shotlines emanate radially from the location of the warhead burst point. The probability of kill of any critical components that lie on any of the radial fragment shotlines needs to be evaluated and the aircraft's vulnerable area and probability of aircraft kill given a hit computed.

There are several approaches to this problem. One simple approach is to expand the presented area of each of the critical components beyond the actual physical size of the component, and then treat a hit by the HE round in the expanded area in the same manner as that used for the nonexplosive penetrator. For example, the presented area of the pilot could be the entire cockpit, because any hit and detonation within the cockpit could kill the pilot. Figure 2.32 illustrates this approach. If the expanded areas of two or more components intersect or overlap, the procedure for accounting for overlapping components described above must be used.

In another procedure, the warhead detonation is assumed to take place at individual locations within a grid superimposed on the presented area of the aircraft, as illustrated in Figure 2.33. Each cell contains one randomly located burst point. The probability of killing the aircraft is then evaluated for each burst point. This kill probability



$$P_{K/H} = \frac{A_V}{A_P} \qquad A_V = \sum_{i=1}^{3} A_{pe_i} * P_{k/h_i}$$

Figure 2.32 The Expanded Area Approach

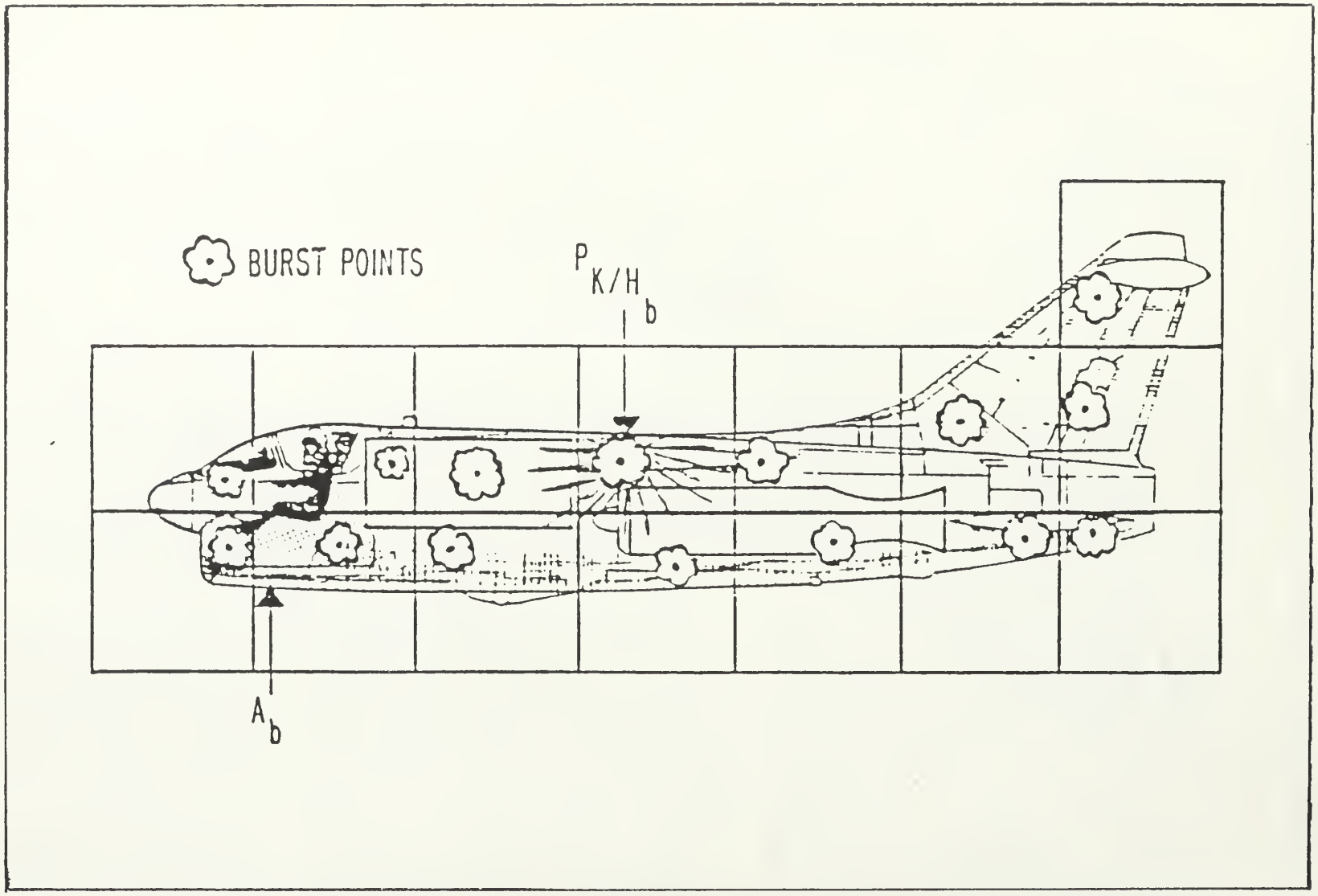


Figure 2.33 The Point Burst Approach

will be dependent upon the relative location of the adjacent critical components and on any shielding of these components provided by intervening structure and non-critical components. Critical components, or parts of critical components, outside of the cell in which the burst occurs must also be considered when they can be hit and killed by the damage mechanisms. Note that several redundant critical components can possibly be killed by the single HE burst. The burst point kill probability is determined using the kill expression for the aircraft. However, because more than one critical component can be killed given a single burst, the individual component kills are not exclusive; a single burst could kill both the fuel system and the pilot. Thus, the approach used in the overlapping component model to compute P_{k/h_o} must also be used here. The probability of an aircraft kill given a random hit from the attack aspect under consideration is obtained by multiplying the probability of aircraft kill given a hit computed for each burst point, P_{K/H_b}, by the probability of a random shot hitting the burst point area, P_{H_b}. The latter probability is given by:

$$P_{H_b} = A_b / A_p , \quad b = 1, 2, \ldots, B \tag{2.70}$$

where B is the number of burst points or cells considered, and A_b is the local grid cell area around each burst point. Note that even though critical components outside of the cell are included in P_{K/H_b}, just the area of the cell itself is used in the computation. The $P_{K/H}$ for the aircraft given a random hit is given by:

$$P_{K/H} = \sum_{b=1}^{B} (P_{H_b})(P_{K/H_b}) = A_p \sum_{b=1}^{B} (A_b)(P_{K/H_b}) \tag{2.71}$$

where A_{v_b} is the vulnerable area of the bth cell.

The aircraft vulnerable area is computed using:

$$A_V = \sum_{b=1}^{B} A_b \left(P_{K/H_b}\right) = \sum_{b=1}^{B} A_{V_b} \qquad (2.72)$$

which is the sum of the vulnerable area of the individual cells.

The vulnerable area for internally detonating HE warheads is usually much larger than the vulnerable area for nonexplosive projectiles and fragments, but it can never exceed the aircraft's presented area.

F. VULNERABILITY TO LASERS

Because a laser beam must hit an aircraft to damage it, and because no high explosive charge is involved, the methodology for assessing the vulnerability of aircraft to lasers consists of essentially the same procedure as used in the assessment of aircraft vulnerability to the single nonexplosive penetrator.

Laser vulnerability is particularly threat sensitive. The first step of the assessment consists of developing a description of the target. Foreign intelligence data and mirror technology are used to describe the target. From this data a computerized target description is generated, allowing the critical components and their failure modes to be identified. The second step of a laser vulnerability study is to accumulate data on the energy density required to produce failure of the critical components, and energy density data on the barrier materials which shield the critical components. From this data, burn through times are calculated as a function of laser beam intensity, power, type of material, and thickness using a parametric penetration equation. A shotline program, using the computerized target description, is used to determine the critical

components and thickness of the shielding material which must be penetrated along each shotline. For each laser dwell time interval, energy is allowed to accumulate, and the time it takes for critical component failure is recorded. An aiming accuracy function is applied for each shotline and the probability of kill, given a laser locked onto target, $P_{K/Lo}(t)$, for each dwell time is calculated.

The general description of laser vulnerability assessment described above applies only to aircraft and ends with $P_{K/Lo}(t)$. Laser vulnerability assessment of a missile is more complicated and takes into account damage produced by the laser to the missile's seeker/guidance system during its trajectory so that the missile misses its target. It should be noted that, for an air defense weapon, the laser is not effective in bad weather conditions where the relative humidity is high.

G. COMPUTER PROGRAMS FOR VULNERABILITY ASSESSMENT

The determination of an aircraft's vulnerability can be a complex and time consuming task. When done manually, many simplifications and assumptions are made, the results are subject to interpretation, and the output is usually limited in scope. Consequently, an extensive number of computer programs or models have been developed by the U.S. military and industry for assessing aircraft vulnerability. These programs can be divided into four major categories; shotline generators, vulnerable area routines, internal burst programs, and Endgame programs. Programs in the first two categories are used for the peretrator and single fragment damage mechanisms. Those in the third category are used for internally detonating HE warheads, and those in the fourth category are for the proximity fuzed HE warhead. (The reader is cautioned that just because a computer is used,

the results are not to be treated as sacrosanct. The output is no more valid than the assumptions that were used to develop the model and the input data.)

Computerized techniques for vulnerability estimates for nonexplosive projectiles and single fragments are currently in wide use. A prerequisite for performing such analyses is the generation of a geometric model of the aircraft describing all of the critical components and the major structure and nonvulnerable components that provide shielding for the critical components. The computer is then programmed to project shotlines (parallel rays) through this model, from selected viewing aspects to provide a sequential listing of penetration data for input to a computerized vulnerability analysis.

1. Shotline Generators

These programs generate shotline descriptions of aircraft targets for use as input data to the codes which calculate vulnerable area. The programs usually model the aircraft external surface and the individual internal and external components either with a set of geometric shapes or with surface patches. The target geometric information required to assess the vulnerability by computer program is generated mathematically by superimposing a planar grid over the target model and by passing a large number of parallel rays through the target from the attack direction to the other side (normal to the grid) through individual grid cells, as shown in Figure 2.34. The position and number of rays is determined by means of the superimposed grid. The number of rays is controlled by selecting the size of the individual squares of the grid. One shotline is randomly located within each cell. Each ray-surface encounter is listed sequentially and identifies the ray location, surface identification number, thickness, obliquity angle, airspace

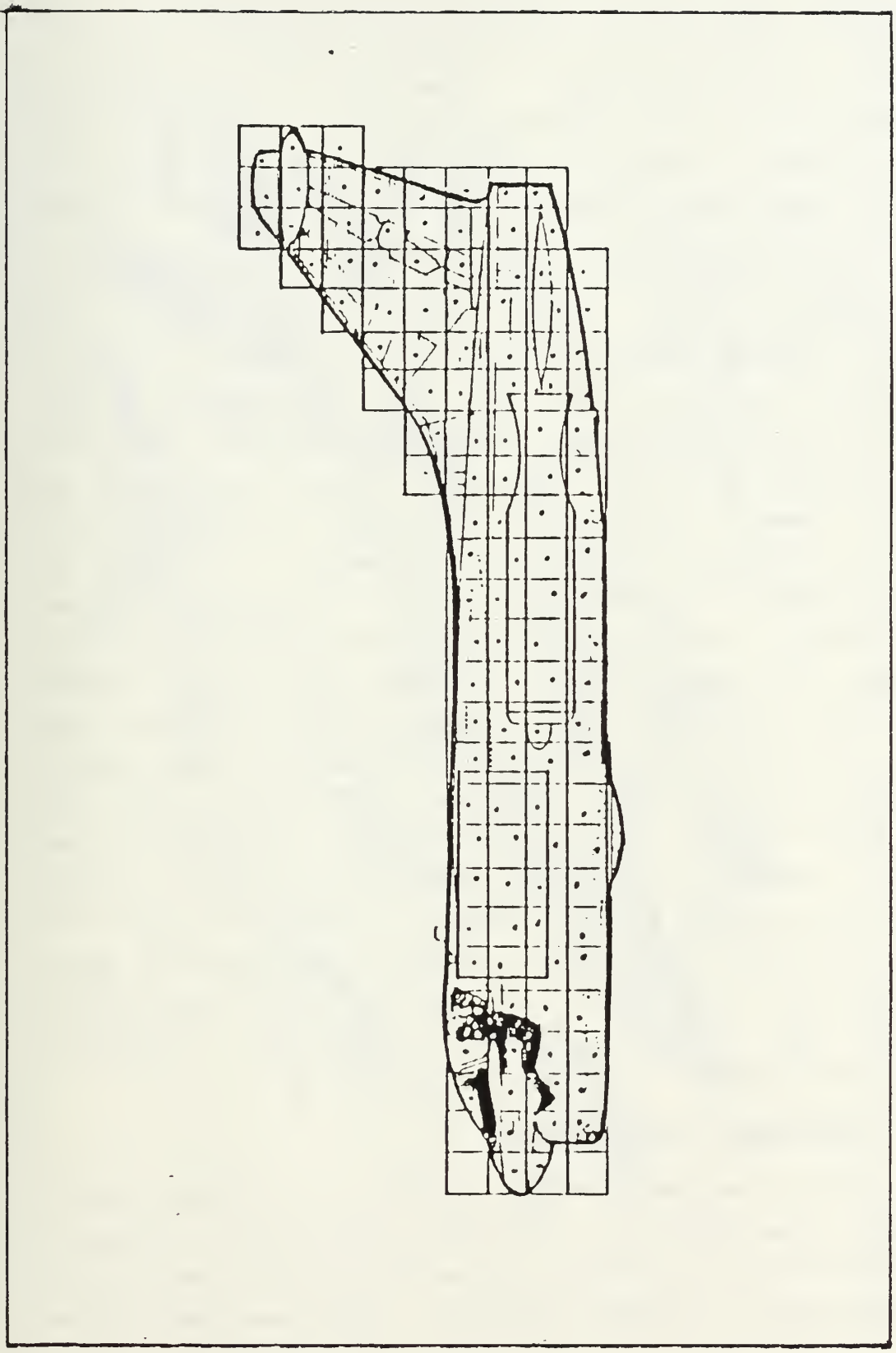


Figure 2.34 Example Grid for the Shotline

encountered, and distance between internal surfaces. This procedure is repeated for all shotlines originating from the selected attack directions. Also the A_p of designated components and of the overall target is output for each viewing aspect. The A_p is approximated by multiplying the number of rays intersecting the target by the area of the individual cells making up the grid plane.

Two families of shotline generator routines have been developed. They are the MAGIC, GIFT family and the SHOTGEN, FASTGEN family. The MAGIC and GIFT codes were develcped at the U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD. These codes use the combinatorial geometry approach, with basic body shapes such as spheres, boxes, cylinders, ellipsoids, and cutting or bounding planes, to describe components. GIFT is an improved version of MAGIC, with simpler input requirements, more efficient computation, and computer-generated graphic displays. The second family, SHOTGEN and the more recent FASTGEN and FASTGEN II, is somewhat similar to the other family, but typically uses the flat triangular patch method to describe the component surfaces. SHOTGEN was developed by the Naval Weapons Center, and FASTGEN and FASTGEN II are improved versions of SHOTGEN sponsored by the Air Force Aeronautical Systems Division (ASD). Figure 2.35 shows the external view of a model built using the combinatorial geometry approach, and Figures 2.36 and 2.37 show the external view and some internal components of a flat triangular surface patch model, respectively.

2. Vulnerable Area Routines

These programs generate component and total aircraft vulnerable area tables for a single penetrator or fragment. The vulnerable area routines can be divided into two groups, the "detailed" or analysis routines, which use the shotline

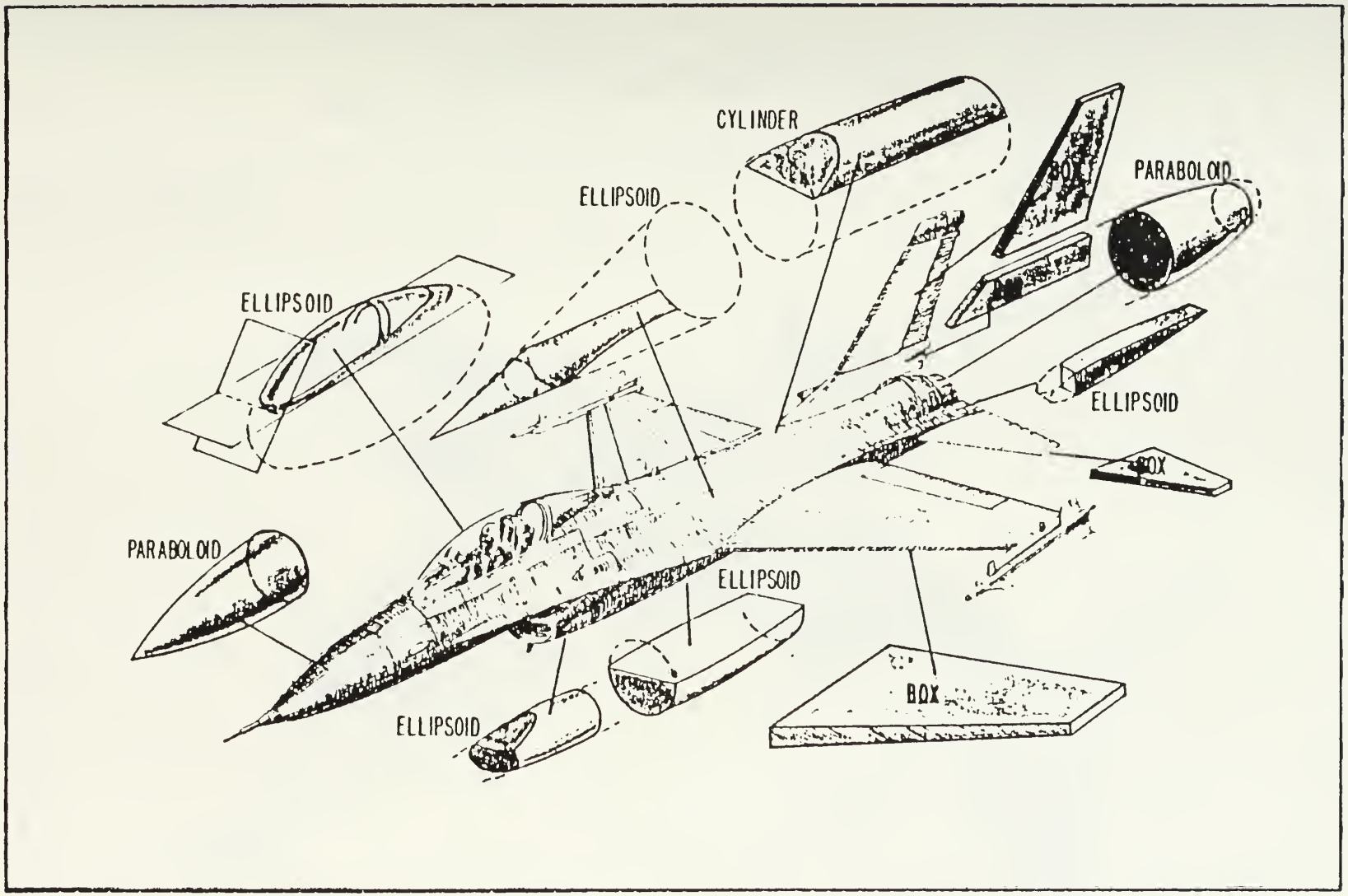


Figure 2.35 Combinatorial Geometry Model of an Aircraft

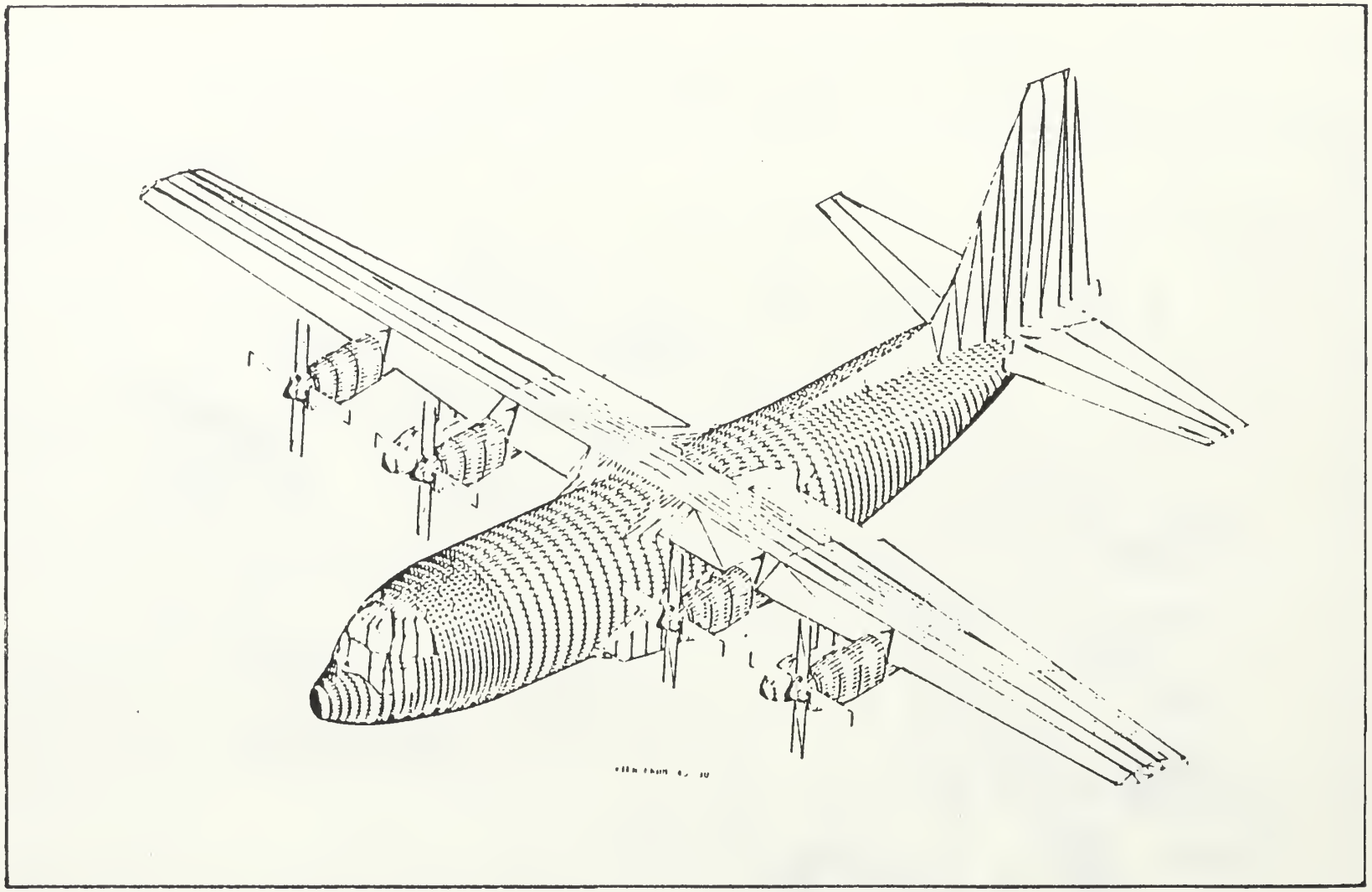


Figure 2.36 External View of Aircraft Using Triangular Surface Patch Method

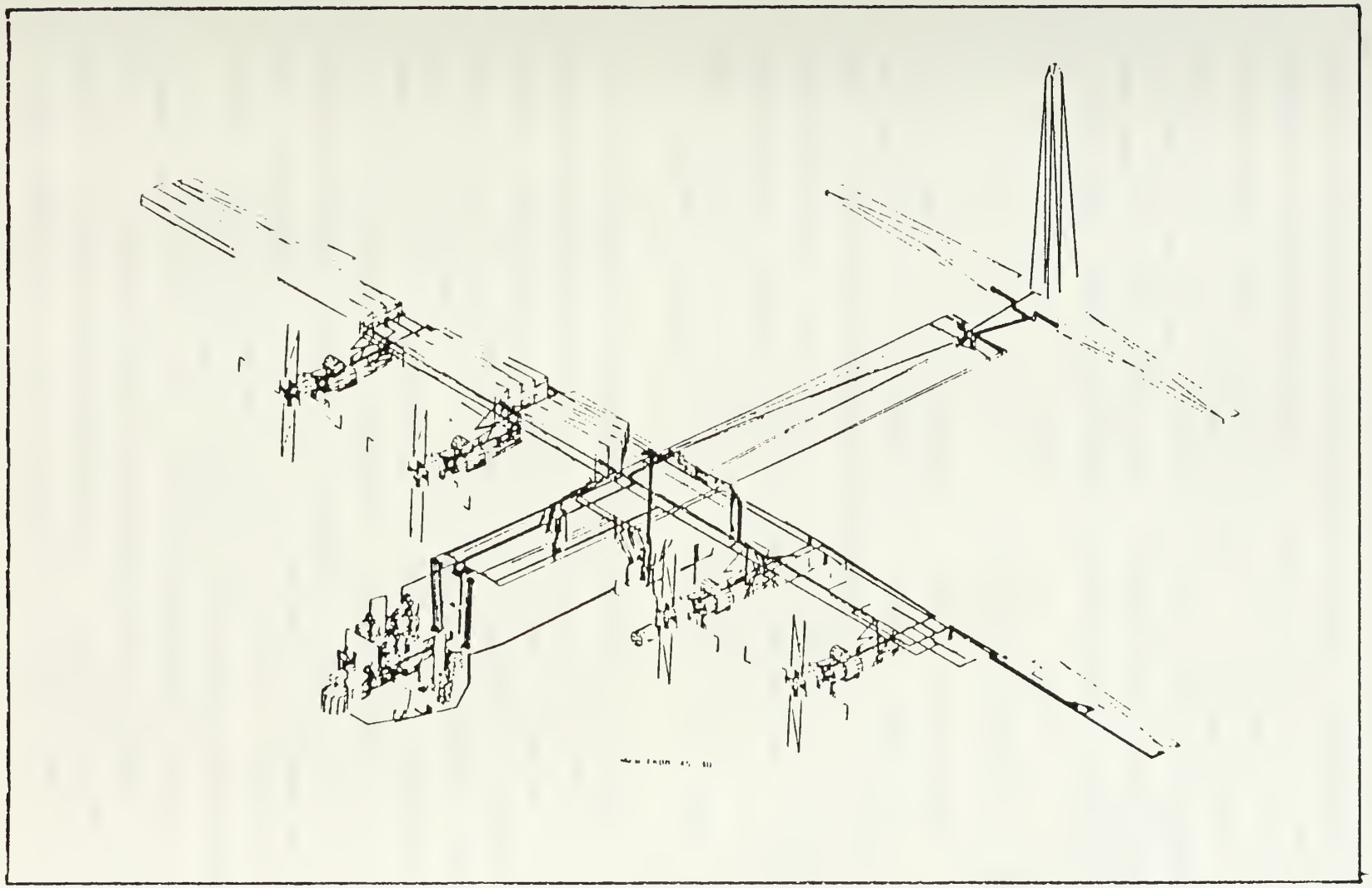


Figure 2.37 Internal Components Modeled Using Triangular Patch Method

approach to compute the vulnerable area, and the "simplified" or evaluation routines which use simplified approaches to determine the vulnerable area. The routines in the analysis group are usually used for problems requiring in-depth studies. However, they have the potential for use in early design studies in which only a limited amount of technical descriptive data is available. The evaluation routines are more appropriate for problems in which a cursory analysis is desired.

a. Analysis Routines

The programs VAREA, VAREA02, and COVART belong to the detailed group. Inputs to these programs include the shotline descriptions of the target model generated by the shotline programs, probability of kill given a hit data for the individual components, empirical ballistic penetration data, and weapon characteristics data. Component and aircraft single hit vulnerable area data are output in tabular form.

VAREA is the oldest and least comprehensive of the three routines in this group. It was developed in 1965 by the Naval Weapons Center to conduct vulnerability analyses of systems subjected to fragmenting-type threats and uses the THOR penetration equations to compute penetrator mass and velocity decay due to penetration through the components along the shotline. VAREA02, completed in 1973, evolved from the VAREA program. Its added capabilities include a projectile penetration mode, an air gap fire model, a redundant components model, and an option to use the DRI penetration equations instead of the THOR relations.

COVART (Computation of Vulnerable Area and Repair Time) currently represents the state-of-the-art in vulnerable area routines. It incorporates all of the features of the VAREA02 program and the helicopter

vulnerable area routines from the HART program and includes a battle damage repair time model. The procedure used by COVART to compute single hit vulnerable areas is essentially the same as that described in this chapter for the single nonexplosive penetrator or fragment. The component vulnerable area of each cell is the product of the cell presented area and the probability of component kill for the shotline in that cell. The vulnerable area of each component is the sum of the component vulnerable areas computed for each grid cell whose shotline passes through the component. The total aircraft vulnerable area is the sum of all of the cell vulnerable areas, considering only the nonredundant critical components and any redundant critical component overlap. Both true and incremental vulnerable areas are available for the overlapping components. Redundant critical components that do not lie along the same shotline do not contribute to the aircraft vulnerability.

b. Evaluation Routines

The computer program COMVAT is representative of the routines which belong to the other group, the simplified codes. These routines were developed to fulfill the need for relatively quick methods for computing vulnerable area. They are intended to be used in situations when use of the more sophisticated routines may not be feasible or timely, such as during early conceptual design studies. The simplified routines are not as accurate as the detailed routines, but they should require considerably less effort and computer run time to use.

COMVAT was developed specifically to compute the vulnerable areas of aircraft components to projectile threats. It is based upon the same principles as the detailed routines, but it does not use shotline descriptions of the aircraft; instead, it computes component vulnerable

areas on the basis of input data describing average shielding conditions on the components. The THOR penetration equations are used to model projectile velocity decay. Secondary effects such as spalling, projectile yawing motions, and projectile break-up are ignored.

3. Internal Burst Programs

Several programs for computing the vulnerability of aircraft to internally detonating HE warheads have been developed under the direction of the Joint Technical Coordinating Group for Munitions Effectiveness (JTCG/ME). These programs are sometimes referred to as point burst programs, and the best known program is the POINTBURST program. This program uses the second approach described in the section on vulnerability to internally detonating HE warheads which is the point burst approach.

4. Endgame Programs

The Endgame refers to the terminal events in an encounter between an aircraft and an HE warhead with a proximity fuze. Just how the warhead got to the vicinity of the aircraft is irrelevant to the Erdgame analysis. The Erdgame events may include target detection by the fuze, and usually do include the warhead detonation, blast propagation, and fragment flyout, impact, and penetration through the aircraft. The numerical value for the $P_{K/D}$ is then determined for the given set of encounter conditions and warhead and aircraft characteristics. This procedure is usually repeated for many different sets of encounter conditions and warhead detonation points, and $P_{K/D}$ is established as a function of the detonation distance. Four Endgame programs currently in use are SESTEM II, SCAN, ATTACK, and REFMCD or MECA. A fifth program, SHAZAM, is nearing completion.

a. SESTEM II

This program was developed in 1977 by the U.S. Air Force Aeronautical Systems Division to evaluate the terminal effectiveness of missiles with nonnuclear blast and fragmentation warheads against aerial targets. The P is computed with respect to a direct hit, fragment damage, and blast. The program has the capability to simulate several fuzing options and a general terminal encounter geometry. The fragment spray angles and density, and fragment average mass, static velocity, cross-sectional area, and coefficient of drag are input data. The target is represented as a collection of shapes that are either single fragment vulnerable, masking, or fuzing components. The external shapes (wing, fuselage, etc.) are modeled using ellipses, and rectangular parallelpipeds are used for the internal components, such as fuel tanks and electronics. The vulnerability of the components is represented by vulnerable area tables. The program can be used to generate iso-$P_{K/D}$ contours.

b. SCAN

SCAN was developed in 1976 under the supervision of the U.S. Navy Pacific Missile Test Center (PMTC) for the Joint Technical Coordinating Group for Aircraft Survivability (JTCG/AS). The objective of SCAN is to predict the probability that an aircraft will survive an attack by a missile armed with a warhead. Aircraft kills due to direct hit, fragment damage, and blast are evaluated. A few fuzing options are considered, as well as a general terminal encounter geometry. The warhead is divided into polar and radial zones and different fragment sizes, shapes, and materials can be specified within each zone. The target is modeled using the combinatorial geometry approach, and component vulnerability to single fragments is expressed by:

$$P_{k/h} = C_1 + [C_2 \times \text{Mass}] + [C_3 \times \text{Velocity}] \qquad (2.73)$$

where mass and velocity refer to the fragment mass and velocity. The energy density and area removal kill criteria are also options for use with components such as major structures. Each component is given a material and thickness and is linked to a subsystem, system, or aircraft kill by a logical kill expression, thus allowing the consideration of redundancy. SCAN also has graphics capabilities for evaluation of the input geometric model and output fragment impact data.

c. ATTACK

ATTACK is a Naval Weapons Center revision of an Endgame methodology developed at the Naval Missile Center, Point Mugu. The object of ATTACK is to predict the ability of a missile to detect and destroy an airborne target. Direct hit, blast, and single fragment (component), and multiple fragment (structural) kills are considered, and a general terminal encounter geometry is provided. The warhead in ATTACK uses the concept of polar and radial fragment spray zones and fragment weight classes. A large number of fuze options are available. The program requires four target models, one for each type of damage, and one fuze model for each encounter. The components in the single fragment model are physically represented by spheres at specified locations, and the vulnerability of each component is contained in vulnerable area tables that depend upon aspect angle, fragment mass, and fragment impact velocity. The multiple fragment model uses a segmented cylindrical target representation, and the vulnerability of each segment is specified by a critical level of fragment energy density.

d. REFMOD (MECA)

The REFMOD program, developed in 1981, was intended to be a reference model to be used for computing the effectiveness of externally detonating weapons against moving targets. (It was later renamed Modular Endgame Computer Analysis or MECA). The model was developed under the auspices of the JTCG/ME Anti-Air Missile Evaluation Group. REFMOD has been assembled by incorporating methodologies from many other existing Endgame programs, including some significant additional features that enable it to work with a wide variety of vulnerability models and to evaluate warhead-target combinations that were previously too cumbersome to assess. The warhead types considered include the continuous rod, divergent fragment spray, convergent fragment spray, focused fragment controlled motion, and an aimable warhead in which the fragment spray density is non-uniform about the missile axis. Several fuze routines are available, and the option exists for the specification of fuzing data from flight tests. The target model and vulnerability employed depend upon the damage mechanism selected. These include direct hit, blast, fragment, and continuous rod. Component vulnerability types for fragments include both vulnerable area and a $P_{k/h}$ kill criterion that is a function of mass, velocity, and density. For the vulnerable area model, components can be described as spherical, linear, cylindrical, or planar in shape, and the component vulnerable area tables generated by COVART can be used. The $P_{k/h}$ vulnerability model employs cylindrical components, and the component kill criterion is given by:

$$P_{k/h} = C_1 \times (\text{Mass})^{C_2} \times (\text{Velocity})^{C_3} \qquad (2.74)$$

with lower and upper threshcld values. By inputting different values for C_1, C_2, and C_3, a variety of kill criteria can be employed. For example, when $C_1 = 0.5$, $C_2 = 1$, and $C_3 = 2$, the energy density criterion is specified.

e. SHAZAM

This code was developed at the Air Force Armament Laboratory (AFATl/DLY) for the evaluation of air-to-air missile effectiveness. The program sequentially assesses the possibility the target aircraft is directly impacted by the missile, the effect of blast overpressures upon the target structures, and the cumulative effect of warhead fragment impacts on the target structure and critical components. The size, shape, and positicn cf the target body and internal components are described by discrete surfaces, and each surface can be vulnerable tc a direct hit, to blast, or to fragments. The criteria used to define the kill of each component/surface are supplied by the user. The program utilizes as much of the aircraft descriptions that are prepared for the SHOTGEN and VAREA programs as is economically feasible. A sufficiently large number of encounter conditions are assessed to generate a single shot probability of kill that has converged to a user specified confidence level.

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